Sections 4.1 & 4.2
Sets. Operations with Sets . Linear Inequalities in One Variable

Example#1 Let A and B be two sets of elements: $A=\{a,b,c\}$, $B=\{a,b,c,d\}$

$a \in A$ because a is an element of A

$d \notin A$ because d is not an element of A.

$\{a,b,c\}=\{b,a,c\}$

Definition $A \subset B$ **A is included in B** if any element of A is also in B.

Example #2 $\{a,b,c\} \subset \{a,b,c,d\}$

$\{1,2,3\} \not\subset \{1,2\}$

Operations with sets

$\cup$ **- "union"** $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Examples:

$\cap$ **- "intersection"** $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Examples:

The Empty Set $\varnothing$ **-** the set with no elements

Definition A number ***a* is less than a number *b*** ($a < b$) if a is to the left of b on the number line.

Exercise #1 Write equivalent statements:

a) $2 \le 3$ _____

b) $2 > y$ _____

c) $5 > x \ge -2$ _____

d) $-4 < -2$ _____

Intervals of real numbers

$[a,b] = \{x \mid a \le x \le b\}$ $(a,b) = \{x \mid a < x < b\}$

$[a,\infty) = \{x \mid x \ge a\}$ $(a,\infty) = \{x \mid x > a\}$

$(-\infty,a] = \{x \mid x \le a\}$ $(-\infty,a) = \{x \mid x < a\}$

Exercise #2 Do the following operations and graph the solution set:

a) $[-2,5] \cup [-3,1]$

b) $[-2,5] \cap [-3,1]$

c) $(1,\infty) \cap (-3,4)$

d) $(-\infty,2) \cup [0,\infty)$

e) $(-4,-1) \cap (-1,2)$

Exercise #3 Graph the following sets and express them using interval notation:

a) $\{x \mid x \le -2\}$

b) $\{x \mid 2 < x \le 3\}$

c) $\{x \mid -3 \ge x \ge -7\}$

Properties of Inequalities

If $a < b$, then $\begin{cases} a + c < b + c \\ a - c < b - c \end{cases}$ for any c.

If $a < b$, then $\begin{cases} ac < bc \\ \frac{a}{c} < \frac{b}{c} \end{cases}$ for any **positive** c.

If $a < b$, then $\begin{cases} ac > bc \\ \frac{a}{c} > \frac{b}{c} \end{cases}$ for any **negative** c.

Exercise #4 (i) Solve each inequality.
(ii) Graph the solution set.
(iii) Write each solution set in interval notation.
(iv) Write each solution set using set notation.

a) $1 + x \le -2$ b) $2 < -3t$ c) $2\left(y + \frac{3}{2}\right) \le 2y + \frac{1}{7}$

d) $\frac{1}{2}(a+8) - \frac{a}{4} > \frac{a}{4}$ e) $-2 < x + 1 \le 5$ f) $-2 < -t + 3 < 5$

g) $15 > 2y - 7 \ge 1$ h) $4(a-1) \ge 3(a-2) + a$ i) $-6 \le -3(x-4) \le 24$

j) $\frac{-(t-3)}{2} + 2 < \frac{3}{4}(2t-5)$ k) $\frac{x-4}{6} \ge \frac{x-2}{9} + \frac{5}{18}$ l) $-3 \le \frac{2x}{3} - 5 < -1$

Exercise #5 (i) Solve each inequality.
(ii) Graph the solution set.
(iii) Write each solution set in interval notation.
(iv) Write each solution set using set notation.

a) $x < -3$ or $x > 3$ b) $x < -3$ and $x > 3$ c) $3x < 3$ or $2x > 10$

d) $3x < 3$ and $2x > 10$ e) $4x + 3 < -1$ or $2x - 3 \ge -11$ f) $4x + 3 < -1$ and $2x - 3 \ge -11$

Exercise #6 A city commission has proposed two tax bills. The first bill requires that a homeowner pay \$1800 plus 3% of the assessed home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal for the homeowner?

Exercise #7 A company manufactures and sells personalized stationery. The weekly fixed cost is \$3000 and it costs \$3.00 to produce each package of stationary. The selling price is \$5.50 per package. How many packages of stationary must be produced and sold each week for the company to have a profit?