Section 5.1 – Introduction to Polynomials and Polynomial Functions

In class work: Complete all statements. Solve all exercises.

Definition A **polynomial** is a finite sum of terms containing whole number exponents on variables.

Definition A **polynomial in *x*** is a finite sum of terms of the form ax^n, where a is a real number and n is a whole number.

1. Give an example of a polynomial (with several terms) in x.

Then, find the following:

- number of terms
- coefficient of each term
- degree of each term
- degree of the polynomial
- leading term
- constant term
- leading coefficient

Definition A **term** is a monomial, that is, an algebraic expression containing a constant or a product between a constant and variables. The constant is called the coefficient of the term.

2. Give one example of each:
   - a monomial in one variable
   - a monomial in three variables
   - a binomial in one variable
   - a binomial of degree three
   - a trinomial in one variable
   - a trinomial of degree two
   - a polynomial of degree 4 in one variable, with a nonzero constant term

Definition The **degree of a nonzero monomial** is the sum of the exponents on the variables.
The **degree of a polynomial** is the greatest degree of its terms
A constant term has a degree of 0 (zero).
The zero term has degree undefined.

Definition A polynomial in one variable is in **standard form** if it is written in descending powers of the variable.

Note

- Polynomial functions of degree 2 or higher have **graphs that are smooth** (no sharp corners) **and continuous** (no breaks).
- The behavior of the graph of a function to the far left (when $x \to -\infty$) or far right (when $x \to \infty$) is called the **end behavior of the graph**.
- The end behavior of a polynomial graph is given by the leading term.

3. Write examples of polynomials and find their end-behavior.

- a polynomial in one variable with odd degree and positive leading coefficient

- a polynomial in one variable with even degree and positive leading coefficient.

- a polynomial in one variable with odd degree and negative leading coefficient

- a polynomial in one variable with even degree and negative leading coefficient.

4. The common cold is caused by a rhinovirus. After *x* days of invasion by the viral particles, the number of particles in our bodies, *f(x)*, in billions, can be modeled by the polynomial function $f(x) = -0.75x^4 + 3x^3 + 5$. Use the leading coefficient test to determine the graph's end behavior to the right. What does this mean about the number of viral particles in our bodies over time?

Exercises 5.1 & 5.2 Operations with polynomial functions

5. If $f(x) = 3x^2$ and $g(x) = -4x^3$, find each new function below and determine the number of the terms of the polynomial function and its degree

    a. $(f+g)(x)$ b. $(fg)(x)$

6. If $f(x) = x^2$ and $g(x) = 3x^2 - 4x + 7$, find each new function below and determine the number of the terms of the polynomial function and its degree

    a. $(f-g)(x)$ b. $(fg)(x)$

7. Do the following operations and simplify as much as possible:

    a. $(-2a^7b)(3a+b)$ b. $(3x-5)(2x+1)$ c. $(3x^n - y^n)(x^n + 2y^n)$

8. If $f(x) = x^2 - 3x + 7$, find each of the following and simplify:

    a) $f(a+2)$ b) $f(a+h) - f(a)$ c) $\dfrac{f(x+h) - f(x)}{h}$ d) $[f(x)]^2$

9. A diver jumps directly upward from a board that is 32 feet high. The function $f(t) = -16t^2 + 16t + 32$ describes the diver's height above the water, *f(t),* in feet, after *t* seconds.

    a) Find and interpret *f(1).* b) Find and interpret *f(2).*

5.4 Multiplication of polynomials. Special products.

Special products	<u>Two terms</u>	<u>Three terms</u>
	$a^2 - b^2 = (a-b)(a+b)$	$a^2 + 2ab + b^2 = (a+b)^2$
	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	$a^2 - 2ab + b^2 = (a-b)^2$
	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	

10. Do the following (simplify). **Use special products.**

$(x+2)^2$	$(3y^3+2)(3y^3-2)$	$(a^n + 4b^n)^2$
$\left(\frac{1}{2}x+3\right)^2$	$\left(\frac{1}{2}x-4y^3\right)^2$	$(10b^n-3)(10b^n+3)$
$(2x-y)^2$	$(x+1)^3$	$(x+y)(x-y)$
$(2x-1)^3$	$(2x-1)(2x+1)$	$(8y+(3x+2))(8y-(3x+2))$