7.1 – 7.6
Radicals and Rational Equations

In-class work:

1) Evaluate each radical. If a radical is not a real number, say so.

$\sqrt{100}$ $\sqrt{0}$ $\sqrt{1}$ $-\sqrt{36}$ $\sqrt{-81}$

$-\sqrt{-1}$ $\sqrt{49+25}$ $\sqrt{49}+\sqrt{25}$ $\sqrt{1.25}\cdot\sqrt{1.25}$ $\left(\sqrt{23}\right)^2$

$\left(-\sqrt{124}\right)^2$ $\sqrt[3]{-27}$ $-\sqrt[3]{-1000}$ $\sqrt[3]{1}$ $\sqrt[3]{-1}$

$\sqrt[4]{16}$ $\sqrt[5]{32}$ $\sqrt[4]{-81}$ $\sqrt{(x+2)^2}$ $\sqrt[3]{(x-3)^3}$

$\sqrt{8^2+6^2}$ $\sqrt{4^2}$ $\sqrt[3]{4^3}$ $\sqrt[5]{x^5}$ $\sqrt{x^{14}}$

$\sqrt[3]{y^{30}}$ $\sqrt{100}-\sqrt[3]{-27}$

2) Graph on the same number line:

$2, \sqrt{5}, -1, 3, \sqrt{8}, 4$

3) For each radical function, find the domain, graph by plotting points, find the range and intercepts.

$f(x)=\sqrt{x}$ $g(x)=\sqrt{x+2}$ $h(x)=\sqrt{x-3}$ $l(x)=\sqrt{1-x}$

$F(x)=\sqrt{x}+2$

4) Find the domain of each radical function:

$f(x)=\sqrt{2x-3}$ $g(x)=\sqrt{4-5x}$ $h(x)=\sqrt[3]{5+6x}$ $l(x)=\dfrac{\sqrt{x-1}}{\sqrt{3-x}}$

5) Use radical notation to rewrite each expression:

$49^{\frac{1}{2}}$ $-16^{\frac{1}{4}}$ $125^{\frac{2}{3}}$ $\left(xy^3\right)^{\frac{1}{7}}$ $2xy^{\frac{2}{3}}$

6) Rewrite each expression with rational exponents:

$\sqrt[3]{x^2}$ $\sqrt[3]{6w^2}$ $\sqrt[5]{x^2y}$ $\sqrt{7}$

7) Simplify . Write the answer using only positive exponents:

$8^{\frac{2}{3}}$ $\quad 49^{-\frac{1}{2}}$ $\quad \left(\frac{8}{27}\right)^{-\frac{1}{3}}$ $\quad (2xy)^{-\frac{7}{10}}$

$3^{\frac{3}{4}} \cdot 3^{\frac{1}{4}}$ $\quad \left(y^{\frac{-2}{3}}\right)^{\frac{1}{4}}$ $\quad \left(2x^{\frac{1}{4}}\right)^{4}$ $\quad \left(x^{\frac{1}{4}} y^{-\frac{2}{5}}\right)^{\frac{1}{3}}$

$\frac{-6y^{-3}}{x^{-3}}$ $\quad \frac{4v^{-5}\left(v^{-2}\right)^{-4}}{3v^{-8}}$ $\quad (a-b)^{-1}$ $\quad (ab)^{-1}$

$\frac{x^3 y^{-2} z}{(xy)^2 z^3}$ $\quad \sqrt[8]{x^2}$ $\quad \sqrt[5]{x^{10} y^{15}}$ $\quad \left(\sqrt[3]{xy}\right)^{18}$

$\sqrt[4]{\sqrt{x}}$ $\quad \left(49x^{-2}y^4\right)^{-\frac{1}{2}}\left(xy^{\frac{1}{2}}\right)$ $\quad \left(\frac{x^{\frac{1}{2}} y^{-\frac{7}{4}}}{y^{-\frac{5}{4}}}\right)^{-4}$

8) Do the following operations. Assume all variables are positive.

$x^2\left(x^5 - xy\right)$ $\quad y^{\frac{1}{2}}\left(y^{\frac{1}{2}} - y^{\frac{1}{3}}\right)$ $\quad \left(x^{\frac{1}{2}} - 3\right)\left(x^{\frac{1}{2}} + 3\right)$ $\quad \left(x^{\frac{1}{3}} - 2\right)\left(x^{\frac{1}{3}} + 6\right)$

9) Factor out the greatest common factor:

$2t^3 - 5t^7$ $\quad 6x^{\frac{1}{2}} + 2x^{\frac{3}{2}}$ $\quad 15y^{\frac{1}{3}} - 60y$

10) Simplify. Assume all variables are positive.

$\sqrt{8}$ $\quad \sqrt[3]{24}$ $\quad \sqrt[3]{27x^3}$ $\quad \sqrt[5]{a^{17}}$

$\sqrt[4]{80x^{10}}$ $\quad \sqrt{12} \cdot \sqrt{2}$ $\quad \sqrt{12x} \cdot \sqrt{3x}$ $\quad \left(-2xy^2\sqrt{3x}\right)\left(xy\sqrt{6x}\right)$

$-3y\left(\sqrt[5]{64x^3y^6}\right)$ $\quad \frac{\sqrt[3]{a^3 - b^3}}{\sqrt[3]{a-b}}$

11) Simplify: $\quad \left[3 + \left(27^{\frac{2}{3}} + 32^{\frac{2}{5}}\right)\right]^{\frac{3}{2}} - 9^{\frac{1}{2}}$

12) Do the following operations. Assume all variables are positive.

$8\sqrt{5} + 3\sqrt{5}$ $\quad 8\sqrt{5}\left(3\sqrt{5}\right)$ $\quad \sqrt{2} + \sqrt{2}$ $\quad \sqrt{2} \cdot \sqrt{2}$

$\sqrt{3}+\sqrt{27}$ $\quad 3\sqrt[3]{24}+\sqrt[3]{81}$ $\quad \dfrac{\sqrt{32}}{5}+\dfrac{\sqrt{18}}{7}$ $\quad \sqrt{2}\left(x+\sqrt{7}\right)$

$\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{2}-\sqrt{7}\right)$ $\quad \left(\sqrt{3}+4\right)^2$ $\quad \left(\sqrt{x}-1\right)^2$ $\quad \left(\sqrt{x-1}\right)^2$

$\left(3\sqrt{x}+1\right)^2$ $\quad \left(5\sqrt{2}+2\sqrt{3}\right)\left(5\sqrt{2}-2\sqrt{3}\right)$ $\quad \dfrac{\sqrt{20}}{3}+\dfrac{\sqrt{45}}{4}-\sqrt{80}$

$\left(\sqrt{5}-\sqrt{10}\right)^2-\left(\sqrt{10}+2\sqrt{5}\right)^2$ $\quad \left(x+\sqrt{6}\right)^2-\left(\dfrac{1}{\sqrt{6}}+x\right)^2$ $\quad \left(\dfrac{\sqrt{2}}{\sqrt{3}}x+y^2\right)\left(\dfrac{\sqrt{2}}{\sqrt{3}}x-y^2\right)$

13) Rationalize each denominator. Assume all variables are positive.

$\dfrac{\sqrt{2}}{\sqrt{5}}$ $\quad \sqrt{\dfrac{11}{3}}$ $\quad \dfrac{9}{\sqrt{3y}}$ $\quad \dfrac{1}{\sqrt[3]{2}}$ $\quad \dfrac{9}{\sqrt{3x^2y}}$

$\dfrac{3}{\sqrt[4]{x}}$ $\quad \dfrac{8}{\sqrt{5}+2}$ $\quad \dfrac{6}{\sqrt{6}-\sqrt{3}}$ $\quad \dfrac{3\sqrt{x}+\sqrt{y}}{\sqrt{y}-3\sqrt{x}}$

14) Rationalize each numerator. Assume all variables are positive.

$\dfrac{\sqrt{x}+3}{\sqrt{x}}$ $\quad \dfrac{\sqrt{x+5}-\sqrt{x}}{5}$ $\quad \dfrac{\sqrt{x}-\sqrt{y}}{x^2-y^2}$

15) Let $f(x)=x^2-6x-4$. Find $f\left(3-\sqrt{13}\right)$.

Answer: 0

16) Let $g(x)=x^2+4x-2$. Find $g\left(-2+\sqrt{6}\right)$.

Answer: 0

17) Find the perimeter and area of a rectangle whose width is $\sqrt{8}-1$ units and length is $\sqrt{8}+1$ units.

Answer: $8\sqrt{2}$ units; 7 square units

18) Let $f(x)=x^2$. Find $f\left(\sqrt{a+1}-\sqrt{a-1}\right)$.

Answer: $2a-2\sqrt{a^2-1}$

19) If $f(x)=\sqrt{5x-1}$, find a such that $f(a)=8$.

20) Solve the following radical equations:

a) $\sqrt{3x+7}+10=4$ b) $x-\sqrt{6x+7}=0$ c) $\sqrt{2t-5}-\sqrt{t+4}=0$

d) $\sqrt[3]{6x-3}=3$ e) $\sqrt{\sqrt{a}+\sqrt{a+8}}=2$ f) $\dfrac{6}{\sqrt{t+5}}=\sqrt{t}$

g) $\sqrt{x}\cdot\sqrt{x+6}=4$ h) $\sqrt{\sqrt{x}+\sqrt{x+9}}=3$

21) Let $g(x)=\sqrt{x-8}-\sqrt{x}$. Find *x* such that $g(x)=-2$.

22) Let $f(x)=\sqrt{x+2}$ and $g(x)=\sqrt{x-1}$.

a) Find $(f+g)(3)$ b) Find *x* such that $(f+g)(x)=3$.

23) Let $h(x)=x+\sqrt{x+5}$.

a) Find $h(7)$ b) Find *x* such that $h(x)=7$.

Answer: a) $7+2\sqrt{3}$; b) 4.

24) Let $f(x)=\sqrt{2x-3}$ and $g(x)=\sqrt{x-2}$.

a) Find $(f-g)(x)$ its domain. b) Find $(f-g)(3)$.

c) Find *x* such that $(f-g)(x)=1$.

Answer: c) 2 , 6.

25) If $l(x)=\sqrt{x+16}-\sqrt{x}-2$, find the domain and the intercepts.

Answer: $[0,\infty];(9,0);(0,2)$

26) In a right triangle, one leg is $\sqrt{x}$ inches and the other leg is $\sqrt{x-7}$ inches. The hypotenuse is $1+\sqrt{x}$ inches.

a) Find the perimeter and area of the triangle in terms of *x*.

b) Find the lengths of the sides of the triangle.

Answer: b) 3, 4, and 5

27) Solve the following equations:

a) $\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)=\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)+2x$ b) $(3x+1)^2=(3x+1)(3x-1)$

c) $4x\sqrt{2}\left(x\sqrt{2}+\sqrt{3}\right)=6x^2+6x\sqrt{6}$

28) Solve each formula for the specified variable:

a) $r=\sqrt{\dfrac{3V}{p\,h}}$ for *V* b) $r=\sqrt{\dfrac{A}{4p}}$ for *A* c) $t=2p\sqrt{\dfrac{l}{32}}$ for *l* d) $v=\sqrt{\dfrac{FR}{m}}$ for *m*