

Highlights Factoring Polynomials

The Greatest Common Factor

Factoring = writing an expression as a product	Factor 12: $12 = 6 \cdot 2$
Prime factorization = writing an expression as a product of prime factors	The prime factorization of 12: $12 = 2^2 \cdot 3$
<div style="display: flex; align-items: center; justify-content: center; gap: 20px;"> <div style="border: 1px solid black; padding: 5px; text-align: center;"> The Greatest Common Factor (GCF) of a list of terms </div> <div style="font-size: 2em;">=</div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> The GCF of numerical coefficients </div> <div style="font-size: 2em;">•</div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> The GCF of the variable factors </div> </div> <p style="text-align: center; margin-top: 10px;"> use prime factorization contains the smallest exponent on each common variable </p>	<p>Find the GCF of $8x^2y$, $12x^3y^2$, and $60x^2y^3$</p> $8x^2y = 2^3 \cdot x^2y$ $12x^3y^2 = 2^2 \cdot 3 \cdot x^3y^2$ $60x^2y^3 = 2^2 \cdot 3 \cdot 5 \cdot x^2y^3$ <p style="text-align: center;">GCF = $2^2 \cdot x^2y = 4x^2y$</p> <p>Factor: $8x + 20 = 4(2x + 5)$ $8x = 2^3 \cdot x$</p>
<p>To factor out the GCF:</p> <ul style="list-style-type: none"> • Find the GCF of the terms • Use the distributive property 	<p>Factor: $20 = 2^2 \cdot 5$ $GCF = 2^2 = 4$ $7(x+2) + y(x+2) = (x+2)(7+y)$ $GCF = x+2$</p>
<p>To factor by grouping:</p> <ul style="list-style-type: none"> • Step 1: Group the terms into two groups of two terms. • Step 2: Factor out the GCF from each group. • Step 3: If there is a common factor, factor it out. • Step 4: If not, rearrange the terms and try Steps 1-3 again. 	<p>Factor: $10x^2 + 15x - 6xy - 9y =$</p> <p>Step 1: $(10x^2 + 15x) - (6xy + 9y) =$ Step 2: $5x(2x + 3) - 3y(2x + 3) =$ Step 3: $(2x + 3)(5x - 3y)$</p>

Special products. Factoring Square Trinomials and the Difference of Two Squares

A perfect square = a positive integer that is the square of a natural number.
This concept of perfect squares extends to algebraic expressions.

A perfect square trinomial = a trinomial that is the square of some binomial .

Squaring a binomial: _____ *Reverse the concept:* **Factoring Perfect Square Trinomials**

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Difference of squares and cubes; Sum of cubes

$$(a - b)(a + b) = a^2 - b^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$(a + b)(a^2 - ab + b^2) = a^3 + b^3$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

To recognize a perfect square trinomial:

- Step1: See if there are two terms that are perfect squares: a^2, b^2
- If no perfect squares, then the trinomial is not a perfect square.
- Step 2: See if the third term can be written as twice the product of a and b: $2ab$

Perfect squares: $1 = 1^2, 4 = 2^2, 9 = 3^2, 16 = 4^2, \dots$

Square each binomial:

$$(x + 6)^2 = x^2 + 2 \cdot x \cdot 6 + 6^2 = x^2 + 12x + 36$$

$$(2x - 3)^2 = (2x)^2 - 2 \cdot (2x) \cdot 3 + 3^2 = 4x^2 - 12x + 9$$

Multiply:

$$(7 - x)(7 + x) = 7^2 - x^2 = 49 - x^2$$

$$(2x - 1)(2x + 1) = (2x)^2 - 1^2 = 4x^2 - 1$$

Factor:

$$x^2 + 6x + 9 = x^2 + 2 \cdot x \cdot 3 + 3^2 = (x + 3)^2$$

Step 1: x^2 and 9 are perfect squares.

Step 2: $6x = 2 \cdot x \cdot 3$

Factor:

$$4x^2 - 12x + 9 = (2x)^2 - 2 \cdot (2x) \cdot 3 + 3^2 = (2x - 3)^2$$

Factor: $x^2 - 25 = x^2 - 5^2 = (x - 5)(x + 5)$

Factoring Trinomials of the Form $Ax^2 + Bx + C$

Step1: Are there any **common factors**? If so, **factor out the GCF**.

- Step2:** Can you recognize any **special products**?

Two terms: Is it the **difference of two squares**?

$$a^2 - b^2 = (a - b)(a + b)$$

Is it a **sum or difference of two cubes**?

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Three terms: Is it a **perfect square trinomial**?

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

- Step3:** Factor using the method that applies, according to the coefficient of x^2

When the coefficient of x^2 is 1 $x^2 + Bx + C$

$$x^2 + Bx + C = (x + ?)(x + ?)$$

*Look after two numbers whose
product = C (the constant term)
sum = B (the coefficient of x)

When the coefficient of x^2 is not 1. $Ax^2 + Bx + C$

$$Ax^2 + Bx + C = Ax^2 + \underset{p}{?}x + \underset{q}{?}x + C$$

*Look after two numbers p and q to rewrite $Bx = px + qx$
whose

product = $A \cdot C$ where A is the coefficient of x^2
C is the constant term

sum = B where B is the coefficient of x

*Then factor by grouping.

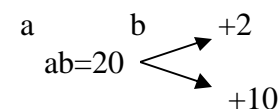
Factor: $x^2 + 12x + 20$

Step1 – there are no common factors.

Step2 – there is no perfect square trinomial.

Step3 – the coefficient of x^2 is 1.

$$x^2 + 12x + 20 = (x + ?)(x + ?) = (x+2)(x+10)$$



$$\begin{array}{l} \underline{a+b=12} \\ 20 = 2 \cdot 10 \end{array}$$

Factor: $6x^2 + 28x - 10$

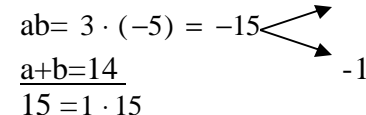
Step1 – the GCF for all terms is 2. Factor 2 out:

$$6x^2 + 28x - 10 = 2(3x^2 + 14x - 5)$$

Step 2 – there is no perfect square trinomial.

Step 3 – the coefficient of x^2 is 3 (not 1).

Look to rewrite $14x = ax + bx = 15x - x$



$$\begin{array}{l} \underline{15 = 1 \cdot 15} \end{array}$$

$$3x^2 + 14x - 5 = 3x^2 + 15x - x - 5$$

Factor by grouping.

$$= 3x(x + 5) - (x + 5) = (x + 5)(3x - 1)$$

So,

$$6x^2 + 28x - 10 = 2(x + 5)(3x - 1)$$

Solving Quadratic Equations by Factoring

A **quadratic equation** is an equation that can be written as

$$Ax^2 + Bx + C = 0, \text{ where } A, B, C \in \mathbb{R}, A \neq 0$$

The form $Ax^2 + Bx + C = 0$ is called the **standard form**.

Quadratic Equations

$$x^2 = 36$$

$$y = -2y^2 + 5$$

Same equations in standard form

$$x^2 - 36 = 0$$

$$2y^2 + y - 5 = 0$$

Zero Factor Property If $a, b \in \mathbb{R}$ and $a \cdot b = 0$, then $a = 0$ or $b = 0$.

If $(x+5)(2x-1) = 0$, then $x+5 = 0$ or $2x-1 = 0$

To Solve Quadratic Equations by Factoring:

- **Step 1:** Write the equation in standard form (one side is zero).
- **Step 2:** Factor completely.
- **Step 3:** Set each factor containing a variable equal to zero.
(according to the Zero Property)
- **Step 4:** Solve the resulting equations.
- **Step 5:** Check solutions in the original equation.

Solve: $3x^2 = 13x - 4$

Step 1 – standard form: $3x^2 - 13x + 4 = 0$

Step 2 – factor (see 4.4) $(3x - 1)(x - 4) = 0$

Step 3 – zero property: $3x - 1 = 0$ or $x - 4 = 0$

Step 4 – solve each linear equation:

$$x = \frac{1}{3} \qquad x = 4$$

Step 5 – check both solutions in the original equation


by replacing x with $\frac{1}{3}$ and then x with 4


Quadratic Equations and Problem Solving


1. Understand the problem.
 - Read and reread it.
 - Draw a diagram.
 - Choose a variable to represent the unknown.
2. Translate the problem into an equation.
3. Solve the equation.
4. Interpret the results: discard the solutions that do not make sense as solutions of the problem. Check your solution in the stated problem and state your conclusion.

Helpful hints:

Perimeter (P) = the sum of the lengths (l) of all sides.

Triangle  $P = l_1 + l_2 + l_3$ $A = \frac{\text{base} \cdot \text{height}}{2}$

Square  $P = 4l$ $A = l^2$

Rectangle  $P = 2l + 2w$ $A = l \cdot w$

Right Triangle - **Pythagorean Theorem:**

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$$

