Math 71

REVIEW
Chapter 1 – The Real Number System

In class work: Solve all exercises.

(Sections 1.1 & 1.2)

Definition **A set** is a collection of objects (elements).

The Set of Natural Numbers $\mathbb{N}$ $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

The Set of Whole Numbers W $W = \{0, 1, 2, 3, 4, 5, \ldots\}$ $\mathbb{N} \subset W$

The Set of Integers $\mathbb{Z}$ $\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

$\mathbb{N} \subset W \subset \mathbb{Z}$

The Set of Rational Numbers $\mathbb{Q}$ $\mathbb{Q} = \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0 \right\}$ $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q}$

The Set of Irrational Numbers Examples: $\sqrt{2}, -\sqrt{5}, p$

The Set of Real Numbers $\mathbb{R}$ $\mathbb{R} = \{x \mid x \text{ is rational or } x \text{ is irrational}\}$

$$\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Mathematical Symbols

SYMBOL	MEANING	EXAMPLES
$=$	is equal to	
$\neq$	is not equal to	
$\in$	belongs to (about an element)	
$\notin$	it doesn't belong to	
$<$	is less than	
$\leq$	is less than or equal to	
$>$	is greater than	
$\geq$	is greater than or equal to	

Properties of Real Numbers

PROPERTIES	ADDITION +	MULTIPLICATION •
COMMUTATIVITY	$a+b=b+a, \quad \forall a,b\in\mathbb{R}$	$ab=ba \quad \forall a,b\in\mathbb{R}$
ASSOCIATIVITY	$(a+b)+c=a+(b+c), \forall a,b,c\in\mathbb{R}$	$(ab)c=a(bc), \quad \forall a,b,c\in\mathbb{R}$
IDENTITY ELEMENT	0 $a+0=0+a=a, \forall a\in\mathbb{R}$	1 $a\cdot 1=1\cdot a=a, \forall a\in\mathbb{R}$
INVERSE ELEMENT	$\forall a\in\mathbb{R}$, there is $-a\in\mathbb{R}$ such that $a+(-a)=(-a)+a=0$	$\forall a\in\mathbb{R}, a\neq 0$, there is $\frac{1}{a}\in\mathbb{R}$ such that $a\cdot\frac{1}{a}=\frac{1}{a}\cdot a=1$
DISTRIBUTIVITY	$a(b+c)=ab+ac$ → multiply out (remove parentheses) ← factor out the common factor	

Exercise #1 Find the opposite and the reciprocal (if any) of each number:

The Number	Its Opposite	Its Reciprocal

The Double Negative Rule

$$-(-a)=a$$

(Section 1.2)

The Absolute Value of a Number

Definition (1) **The absolute value of a number** is the distance between the number and 0 (the origin) on the number line.

$$|a| = dist(a,0)$$

Property $|a| \geq 0, \quad \forall a \in R$

Definition (2)

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Properties

(1) $|ab| = |a| \cdot |b|, \ \forall a, b \in \mathbb{R}$

(2) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \forall a, b \in \mathbb{R}, b \neq 0$

Note: $|a+b| \neq |a| + |b|$ Example: ______________________

$|a-b| \neq |a| - |b|$ Example: ______________________

Exercise #2 Simplify the following:

a) $|-7| =$

b) $-(-7) =$

c) $-|-7| =$

d) $-|-(-7)| =$

Exercise #3 Simplify the following:

a) $(-5)^2 - 3^2 + |10 - 2 \cdot 3|$ (A: 20)

b) $\dfrac{(-4)^2 - |1 - 2^3|}{-(-2)^3 + (-1)^{125}}$ (A: $\frac{9}{7}$)

c) $\dfrac{9[4-(1+6)]-(3-9)^2}{5+\dfrac{12}{5-\dfrac{6}{2+1}}}$ (A: -7)

d) $-\dfrac{4}{3} - \dfrac{9}{4} + \dfrac{11}{6}$ (A: -7/4)

e) $\left(\dfrac{3}{20} - \dfrac{5}{24}\right)\left(\dfrac{5}{6} - \dfrac{1}{21}\right)$ (A: -11/240)

Exercise #4 Evaluate the following expressions if $x = 2, y = -3, z = -1$:

a) $\dfrac{3y^2 - x^2 + 1}{y|z|}$ (A: -8)

b) $yz^3 - (xy)^3$ (A: 219)

(Section 1.6)

Properties of Integral Exponents

<u>Definition</u> If $n \in \mathbb{N}$, then $a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$

a is called **base** and *n* is called **power (exponent).**

PROPERTY		EXAMPLES
The Product Rule	$a^m \cdot a^n = a^{m+n}$	
The Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$	
The Zero-Exponent Rule	$a^0 = 1, \forall a \neq 0$	
The Negative-Exponent Rule	$a^{-n} = \frac{1}{a^n}$	
The Power Rule	$\left(a^m\right)^n = a^{m \cdot n}$	
Products to Power	$(ab)^n = a^n \cdot b^n$	
Quotients to Power	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

Exercise #5 Simplify the following expressions:

a) $x-5[x-5(x-5)]$ $(A\text{: } 21x-125$

b) $x^2y(xy-x)-7xy(x^2y-x^2)$ $\left(A:-6x^3y^2+6x^3y\right)$

c) $(-8xy)(x^5y^4)(-4xy)$ $\left(A:32x^7y^6\right)$

d) $x\left[2x^2+x\left(x-3\left(x-1\right)\right)\right]$ $\left(A:3x^2\right)$

e) $2\left(x-1\right)\left(3x+2\right)-5\left(2-x\right)\left(2x+3\right)$

f) $7x-4\left[2x-5\left(x+3\right)-1\right]$ $\left(A:19x+64\right)$

g) $\frac{2}{3}x+\frac{1}{3}y-x+\frac{2}{6}y-\frac{3}{4}x$ $\left(A:-\frac{13}{12}x+\frac{2}{3}y\right)$

Exercise #6 Simplify each expression .
Write answers without using parentheses or negative exponents.

a) $\frac{y^2}{yy^{-2}}$ $\left(A:y^3\right)$

b) $\left(\frac{a^2b^{-1}}{4a^3b^{-2}}\right)^{-3}$ $\left(A:\frac{64a^3}{b^3}\right)$

c) $\frac{a^0+b^0}{2\left(a+b\right)^0}$ $\left(A:1\right)$

d) $\left(\frac{-2a^{-4}b^3c^{-1}}{3a^{-2}b^{-5}c^{-2}}\right)^{-4}$ $\left(A:\frac{81a^8}{16b^{32}c^4}\right)$

e) $\left(\frac{2x^{-4}y}{x^5y^5}\right)^{-3}\left(\frac{4x^{-2}y^0}{x^7y^2}\right)^2$ $\left(A:2x^9y^8\right)$

f) $\frac{24x^2y^{13}}{-2x^5y^{-2}}$ $\left(A:-\frac{12y^{15}}{x^3}\right)$

g) $\left(-4x^{-4}y^5\right)^{-2}\left(-2x^5y^{-6}\right)$ $\left(A:-\frac{x^{13}}{8y^{16}}\right)$

Exercise #7

a) Find the set $A=\left\{x\middle|x\in Z,-3\le x<2\right\}$.

b) Find the set $B=\left\{x\middle|x\in N,\sqrt{10}<x\le\sqrt{25}\right\}$

Exercise #8

a) Find x such that $\frac{3}{x}\in N, x\in Z$.

b) Find x such that $\frac{15}{3x+2}\in Z, x\in Z$.

(Sections 1.4 & 1.5)

Linear Equations

Definition An **equation** is a mathematical statement that two algebraic expressions are equal.

Examples:

Types of Equations

(1) **IDENTITY** = an equation which is always **true** regardless of the value of the variable.

Examples: $3 = 3$

$x + 1 = x + 1$

(2) **CONTRADICTION** = an equation which is always **false** regardless of the value of the variable.
(INCONSISTENT)

Examples: $5 = 7$

$x + 2 = x + 4$

(3) **CONDITIONAL** = an equation whose truth or falsehood depends on the value of the variable.

Examples: $x + 2 = 5$

Exercise #10 Determine the type of each of the following equations:

a) $2(x-3) = 2x - 3$

b) $5(x+2) = 5x + 10$

c) $3(w+1) = w + 3$

Definition A **solution** of an equation is the value of the variable that **satisfies** the equation.

Definition The process of finding the values that satisfy an equation is called **solving the equation.**

Exercise #11 Determine which of the listed values satisfies the given equation:

a) $2x+3=6$, $x=0, x=\frac{3}{2}$

b) $6-2w=10-3w$, $w=-4, w=1$

Properties of Equality

$$\text{If } a=b\text{, then } \begin{cases} a+c=b+c, \forall c\in\mathbb{R} \\ a-c=b-c, \forall c\in\mathbb{R} \\ ac=bc, \forall c\in\mathbb{R} \\ \frac{a}{c}=\frac{b}{c}, \forall c\neq 0 \end{cases}$$

Exercise #12 Solve the following equations .

a) $x-4=8$

b) $a+15=15$

c) $-6=-x+21$

d) $6x=5$

e) $10t=-36$

f) $2a=0$

g) $\frac{1}{5}p=-3$

h) $-9x=18$

j) $-x=-\frac{3}{4}$

k) $-\frac{3}{5}t=\frac{2}{7}$

l) $2a+3=4$

m) $-4x-1=5$

n) $3x+1=x+2$

o) $-\frac{2}{7}z+2=\frac{5}{7}z$

q) $5.6t+2=4.6t$

p) $5x+4-4x=0$

r) $6x+5+7x+3=12x+4$

s) $2(p+5)-(9+p)=-3$

t) $2(y+4)-2y=8$

u) $5x+8=2x+8$

v) $-3\left(x-\frac{1}{4}\right)=-4x$

x) $\frac{q}{2}+13=-22$

y) $3-3(5-t)=0$

z) $(3-3)(5-x)=0$

w) $7a-5(a-2)-a=4a-2(a-5)-a$

a) $4x-3(x+8)=5x-2(x-12)-2x$

Exercise #13 Solve the following equations .

a) $\frac{3}{4}z-\frac{1}{4}=\frac{3}{4}$

b) $\frac{4}{5}y-\frac{1}{5}=\frac{3}{5}$

c) $\frac{x+4}{2}+\frac{x+1}{4}=3$

d) $\frac{w+3}{6}-\frac{w+4}{2}=2$

e) $\frac{2}{3}(v-4)=2$

f) $\frac{3}{4}(u-6)=2$

g) $\frac{5}{3}(t-1)=\frac{4}{5}(2t+1)+\frac{2}{3}$

h) $\frac{4}{5}(s+2)=\frac{1}{2}+\frac{5}{6}(s+3)$

i) $\frac{3(n-2)}{5}=\frac{3n+6}{6}$

j) $\frac{x}{3}+\frac{1}{6}=\frac{2}{5}$

k) $\frac{6}{7}m-\frac{3}{4}=\frac{4}{5}-\frac{1}{7}m+\frac{1}{6}$

l) $\frac{2}{3}k-\left(k-\frac{1}{2}\right)=\frac{1}{6}(k-51)$

m) $\frac{1}{2}(x-1)-\frac{3}{4}x+5=\frac{1}{6}$ n) $\frac{1}{3}(x+3)+\frac{1}{6}(x-6)=x+3$

o) $-\frac{5}{6}q-(q-1)=\frac{1}{4}(-q+80)$ p) $-\frac{1}{4}(x-12)+\frac{1}{2}(x+2)=x+4$

Exercise #14 Solve the following equations .

a) $0.8q-3.2=1.6$ b) $2.3r-4.7=4.5$ c) $2.3s+4.7s=4.9$

d) $5.1m+2.3m=2.96$ e) $0.4(0.2n-0.3)=0.01$ f) $0.8(0.3p-0.5)=0.8$

g) $0.8q-0.3(210-q)=80$ h) $0.3r+1.2(20)=0.8(r+20)$ i) $x+0.05(12-x)=0.1(63)$

j) $3(y-0.87)-2y=4.98$ k) $0.4y+0.3(20-y)=0.1y+6$ l) $0.1x+0.05(x-300)=105$

Exercise #15 Solve the following equations .

a) $15\%q=6$ b) $30\%r=9$ c) $50\%s+s=12$

d) $75\%t+t=105$ e) $20\%u+25\%u=18$ f) $50\%t+20\%(90-t)=30$

g) $20\%+40\%(25-s)=9$

Answers #13: a) 4/3; b) 1; c) 1; d) -21/2;e) 7; f) 26/3; g) 47; h) -42; i) 22 ; m) 52/3; o) -12
Answers #14: a) 6; b) 4; c) .7; d) .4; e) 13/8; f) 5; g) 130; h) 16
Answers #15: a) 40; b) 30; c) 8; d) 60; e) 40; f) 40; g) 3

Exercise #16

Evaluate $x^2-(xy-y)$ for x satisfying $\frac{3(x+3)}{5}=2x+6$ and y satisfying $-2y-10=5y+18$.

Exercise #17 Solve each formula for the specified variable:

a) $v=k+gt$, for t $\left(A: t=\frac{v-k}{g}\right)$

b) S=3pd+pa, for d $\left(A: d=\frac{S-pa}{3p}\right)$

c) $A=P(1+rt)$, for r $\left(A: r=\frac{A-p}{pt}\right)$

d) $A=2w^2+4lw$, for l $\left(A: l=\frac{A-2w^2}{4w}\right)$

e) $A=\frac{1}{2}h(a+b)$ for a $\left(A: a=\frac{2A}{h}-b\right)$

f) $A=2lw+2lh+2wh$ for l $\left(A: l=\frac{A-2wh}{2(w+h)}\right)$

Chapter 5
Review of Factoring Expressions 5.3 – 5.6

Note: Factoring an expression changes it from a *sum* into a *product.*

I Factoring The Greatest Common Factor (5.3)

This is a direct application of the *distributive property*: $ab+ac=a(b+c)$

Exercise # 18 Factor as completely as possible:

1. $8x^3+20x-28$
2. $6x^2-12x$
3. $24a^3b-36a^2c^2+48ab^3$
4. $15x^3y^4-5x^2y^3$
5. $-20xy^3+35x^2y-60xy$
6) $9m-12n+8p$
7) $2t^2+8t$
8) x^2-x
9) $a(a+7)+3(a+7)$
10) x^2y-xy^2
11) $z(z-3)-5(z-3)$
12) $\frac{1}{4}d^2-\frac{3}{4}d$
13. $c(b+9)+2(b+9)$
14. $s(4r-3)-t(4r-3)$
15. $8x(y-2)+(y-2)$
16. $3z(4b+1)-(4b+1)$

II Factoring by Grouping (5.3)

We use this method when we usually have four or more terms.

Exercise # 19 Factor as completely as possible:

1) $x^2+4x+xy+4y$
2) $a^2-ab-3a+3b$
3) $m^2+mn+9m+9n$
4) $x^2-xy-4x+4y$
5) $y^2+5y-7y-35$
6) $1-a+ab-b$
7) $16m^3-4m^2p^2-4mp+p^3$
8) $7z^2+14z-2z-4$
9) $10x^2-15x-2x+3$
10) $8pq+12p+10q+15$
11) $3a^3-21a^2b-2ab+14b^2$
12) $8u^2v^2+16u^2v+10uv^2+20uv$

III Special Products (5.5)

Two Terms

Difference of Squares: $a^2-b^2=(a-b)(a+b)$

Sum of Squares: a^2+b^2 - not factorable

Difference of Cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Sum of Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Three Terms

Perfect Square Trinomials

$a^2+2ab+b^2=(a+b)^2$

$a^2-2ab+b^2=(a-b)^2$

<u>Exercise # 20</u> Factor as completely as possible:

1) $x^2 - 25$
2) $p^2 - \frac{1}{9}$
3) $w^2 + 2w + 1$
4) $a^2 + 4a + 4$
5) $100x^2 + 49$
6) $m^3 - 8$
7) $64y^3 - 27$
8) $6x^3 + 6$
9) $16x^2 - 40x + 25$
10) $x^4 - 1$
11) $y^8 - 256$
12) $x^9 + y^9$
13) $\frac{36}{25} - b^2$
14) $2n^2 - 288$
15) $6c^3 + 48$
16) $p^6 - 1$
17) $\frac{16}{9}x^2 - \frac{1}{49}$
18) $(d+4)^2 - (d-3)^2$
19) $(x+7)^3 + 8$

<u>IV Factoring Trinomials $ax^2 + bx + c$ (5.4)</u>

<u>Case 1 Leading coefficient is1:</u> $a = 1$

Factor $x^2 + bx + c = (x \; \square)(x \; \square)$

product = c
sum = b

<u>Exercise # 21</u> Factor as completely as possible:

1) $x^2 + 5x + 6$
2) $x^2 - 5x - 6$
3) $x^2 - 7x + 6$
4) $x^2 - x - 6$
5) $2a^2 + 8a + 10$
6) $2x + x^2 - 15$
7) $6t^2 - 18t + 12$
8) $z^2 - 17z + 30$
9) $-x^2 - 15x - 36$
10) $m^2 + 6m - 18$
11) $x^2 - 3xy + 2y^2$
12) $t^2 - tz - 6z^2$
13) $24 + 14d + d^2$
14) $x^2 - 7x - 15$
15) $3w^2 - 12w - 96$
16) $-w^2 - 2w + 3$

<u>Case 2 Leading coefficient is not 1</u>: $a \neq 1$

Factor $ax^2 + bx + c = ax^2 + \square x + \square x + c$ by splitting the middle term bx then using grouping

product = ac
sum = b

Factor as completely as possible:

17) $6x^2 + 7x - 20$
18) $2t^2 - 7t + 3$
19) $6a^2 + 40a + 24$
20) $3x^2 - 11x - 20$
21) $15 + 6b^2 - 19b$
22) $25y^2 + 35y + 45$
23) $7x - x^2 - 10$
24) $-6p^2 + 28p - 32$
25) $8a^2 + 23ab - 3b^2$
26) $2r^2 + 9r + 10$
27) $7 + 18u + 8u^2$
28) $20b^2 - 32b - 45$
29) $-7a^2 + 4a + 3$
30) $13x^2 + 17x - 18$
31) $45q^2 + 57q + 18$
32) $-16k^3 + 48k^2 - 36k$
33) $14k^3 + 7k^2 - 70k$