REVIEW
Chapter 1 – The Real Number System

> **In class work** : Solve all exercises.

(Sections 1.1 & 1.2)

<u>Definition</u>   **A set** is a collection of objects (elements).

<u>The Set of Natural Numbers</u> $\mathbb{N}$ $\mathbb{N} = \{ \text{ 1, 2, 3, 4, 5,...} \}$

<u>The Set of Whole Numbers</u> **W** $W = \{ \text{ 0, 1, 2, 3, 4, 5,...} \}$ $\mathbb{N} \subset W$

<u>The Set of Integers</u> $\mathbb{Z}$ $\mathbb{Z} = \{ \text{ ..., -4, -3, -2, -1 ,0, 1, 2, 3, 4, 5,...} \}$

$\mathbb{N} \subset W \subset \mathbb{Z}$

<u>The Set of Rational Numbers</u> $\mathbb{Q}$ $\mathbb{Q} = \left\{ \dfrac{a}{b} \middle| a,b \in \mathbb{Z}, b \neq 0 \right\}$ $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q}$

<u>The Set of Irrational</u> Numbers   Examples: $\sqrt{2}, -\sqrt{5}, \boldsymbol{p}$

<u>The Set of Real Numbers</u> $\mathbb{R}$ $\mathbb{R} = \{ x \mid x \text{ isrational or } x \text{ is irrational} \}$

$$\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Mathematical Symbols

SYMBOL	MEANING	EXAMPLES
$=$	is equal to	
$\neq$	is not equal to	
$\in$	belongs to (about an element)	
$\notin$	it doesn't belong to	
$<$	is less than	
$\leq$	is less than or equal to	
$>$	is greater than	
$\geq$	is greater than or equal to	

Properties of Real Numbers

PROPERTIES	ADDITION +	MULTIPLICATION $\bullet$
COMMUTATIVITY	$a+b=b+a, \quad \forall a,b \in \mathbb{R}$	$ab=ba \quad \forall a,b \in \mathbb{R}$
ASSOCIATIVITY	$(a+b)+c = a+(b+c), \forall a,b,c \in \mathbb{R}$	$(ab)c = a(bc), \quad \forall a,b,c \in \mathbb{R}$
IDENTITY ELEMENT	0 $a+0=0+a=a, \forall a \in \mathbb{R}$	1 $a \cdot 1 = 1 \cdot a = a, \forall a \in \mathbb{R}$
INVERSE ELEMENT	$\forall a \in \mathbb{R}$, there is $-a \in \mathbb{R}$ such that $a+(-a)=(-a)+a=0$	$\forall a \in \mathbb{R}, a \neq 0$, there is $\dfrac{1}{a} \in \mathbb{R}$ such that $a \cdot \dfrac{1}{a} = \dfrac{1}{a} \cdot a = 1$
DISTRIBUTIVITY	$a(b+c)=ab+ac$ $\xrightarrow{\hspace{2cm}}$ multiply out (remove parentheses) $\xleftarrow{\hspace{2cm}}$ factor out the common factor	

Exercise #1   Find the opposite and the reciprocal (if any) of each number:

The Number	Its Opposite	Its Reciprocal

The Double Negative Rule

$$-(-a)=a$$

(Section 1.2)

The Absolute Value of a Number

<u>Definition (1)</u> **The absolute value of a number** is the distance between the number and 0 (the origin) on the number line.

$$|a| = dist(a,0)$$

<u>Property</u> $|a| \geq 0, \quad \forall a \in R$

<u>Definition (2)</u> $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$

<u>Properties</u>   (1) $|ab| = |a| \cdot |b|, \forall a, b \in \mathbb{R}$

(2) $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \forall a, b \in \mathbb{R}, b \neq 0$

Note: $|a+b| \neq |a| + |b|$ Example: _____

$|a-b| \neq |a| - |b|$ Example: _____

<u>**Exercise #2**</u>   Simplify the following:

a) $|-7| =$ c) $-|-7| =$

b) $-(-7) =$ d) $-|-(-7)| =$

<u>**Exercise #3**</u>   Simplify the following:

a) $(-5)^2 - 3^2 + |10 - 2 \cdot 3|$ (A: 20)

b) $\dfrac{(-4)^2 - |1 - 2^3|}{-(-2)^3 + (-1)^{125}}$ (A: $\dfrac{9}{7}$)

c) $\dfrac{9[4 - (1+6)] - (3-9)^2}{5 + \dfrac{12}{5 - \dfrac{6}{2+1}}}$ (A: -7)

d) $-\dfrac{4}{3} - \dfrac{9}{4} + \dfrac{11}{6}$ (A: -7/4)

e) $\left(\dfrac{3}{20} - \dfrac{5}{24}\right)\left(\dfrac{5}{6} - \dfrac{1}{21}\right)$ (A: -11/240)

<u>**Exercise #4**</u>   Evaluate the following expressions if $x = 2, y = -3, z = -1$:

a) $\dfrac{3y^2 - x^2 + 1}{y|z|}$ (A: -8)

b) $yz^3 - (xy)^3$ (A: 219)

Properties of Integral Exponents

<u>Definition</u>   If $n \in \mathbb{N}$, then $a^n = a \cdot a \cdot ... \cdot a$
                                                  n times
            a is called **base** and n is called **power (exponent).**

PROPERTY		EXAMPLES
The Product Rule	$a^m \cdot a^n = a^{m+n}$	
The Quotient Rule	$\dfrac{a^m}{a^n} = a^{m-n}$	
The Zero-Exponent Rule	$a^0 = 1, \forall\, a \neq 0$	
The Negative-Exponent Rule	$a^{-n} = \dfrac{1}{a^n}$	
The Power Rule	$\left(a^m\right)^n = a^{m \cdot n}$	
Products to Power	$(ab)^n = a^n \cdot b^n$	
Quotients to Power	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	

Exercise #5   Simplify the following expressions:

a) $x-5[x-5(x-5)]$ $(A: 21x-125$

b) $x^2 y(xy-x)-7xy(x^2 y-x^2)$ $\left(A:-6x^3 y^2+6x^3 y\right)$

c) $(-8xy)(x^5 y^4)(-4xy)$ $\left(A:32x^7 y^6\right)$

d) $x\left[2x^2+x\left(x-3(x-1)\right)\right]$ $\left(A:3x^2\right)$

e) $2(x-1)(3x+2)-5(2-x)(2x+3)$

f) $7x-4\left[2x-5(x+3)-1\right]$ $(A:19x+64)$

g) $\dfrac{2}{3}x+\dfrac{1}{3}y-x+\dfrac{2}{6}y-\dfrac{3}{4}x$ $\left(A:-\dfrac{13}{12}x+\dfrac{2}{3}y\right)$

Exercise #6   Simplify each expression .
Write answers without using parentheses or negative exponents.

a) $\dfrac{y^2}{yy^{-2}}$ $\left(A:y^3\right)$

b) $\left(\dfrac{a^2 b^{-1}}{4a^3 b^{-2}}\right)^{-3}$ $\left(A:\dfrac{64a^3}{b^3}\right)$

c) $\dfrac{a^0+b^0}{2(a+b)^0}$ $(A:1)$

d) $\left(\dfrac{-2a^{-4}b^3 c^{-1}}{3a^{-2}b^{-5}c^{-2}}\right)^{-4}$ $\left(A:\dfrac{81a^8}{16b^{32}c^4}\right)$

e) $\left(\dfrac{2x^{-4}y}{x^5 y^5}\right)^{-3}\left(\dfrac{4x^{-2}y^0}{x^7 y^2}\right)^2$ $\left(A:2x^9 y^8\right)$

f) $\dfrac{24x^2 y^{13}}{-2x^5 y^{-2}}$ $\left(A:-\dfrac{12y^{15}}{x^3}\right)$

g) $\left(-4x^{-4}y^5\right)^{-2}\left(-2x^5 y^{-6}\right)$ $\left(A:-\dfrac{x^{13}}{8y^{16}}\right)$

Exercise #7

a) Find the set $A=\{x|x\in Z,-3\le x<2\}$.

b) Find the set $B=\{x|x\in N,\sqrt{10}<x\le\sqrt{25}\}$

Exercise #8

a) Find x such that $\dfrac{3}{x}\in N, x\in Z$.

b) Find x such that $\dfrac{15}{3x+2}\in Z, x\in Z$.

(Sections 1.4 & 1.5)

Linear Equations

<u>Definition</u>   An **equation** is a mathematical statement that two algebraic expressions are equal.

   <u>Examples:</u>

<u>Types of Equations</u>

   (1) **IDENTITY** = an equation which is always **true** regardless of the value
                   of the variable.

            <u>Examples:</u> $3 = 3$

                  $x + 1 = x + 1$

   (2) **CONTRADICTION** = an equation which is always **false** regardless of the
        (INCONSISTENT)   value of the variable.

            <u>Examples:</u> $5 = 7$

                  $x + 2 = x + 4$

   (3) **CONDITIONAL** = an equation whose truth or falsehood depends on the value of
                   the variable.

            <u>Examples:</u> $x + 2 = 5$

<u>Exercise #10</u> Determine the type of each of the following equations:
         a) $2(x - 3) = 2x - 3$
         b) $5(x + 2) = 5x + 10$
         c) $3(w + 1) = w + 3$

<u>Definition</u>   A **solution** of an equation is the value of the variable that **satisfies** the equation.

<u>Definition</u>   The process of finding the values that satisfy an equation is called **solving the equation.**

Exercise #11 Determine which of the listed values satisfies the given equation:

a) $2x+3=6$, $x=0, x=\dfrac{3}{2}$

b) $6-2w=10-3w$, $w=-4, w=1$

Properties of Equality

If $a=b$, then $\begin{cases} a+c=b+c, \forall c\in \mathbb{R} \\[2mm] a-c=b-c, \forall c\in \mathbb{R} \\[2mm] ac=bc, \forall c\in \mathbb{R} \\[2mm] \dfrac{a}{c}=\dfrac{b}{c}, \forall c\neq 0 \end{cases}$

Exercise #12 Solve the following equations .

a) $x-4=8$

b) $a+15=15$

c) $-6=-x+21$

d) $6x=5$

e) $10t=-36$

f) $2a=0$

w) $7a-5(a-2)-a=4a-2(a-5)-a$

g) $\dfrac{1}{5}p=-3$

h) $-9x=18$

j) $-x=-\dfrac{3}{4}$

k) $-\dfrac{3}{5}t=\dfrac{2}{7}$

l) $2a+3=4$

m) $-4x-1=5$

n) $3x+1=x+2$

o) $-\dfrac{2}{7}z+2=\dfrac{5}{7}z$

q) $5.6t+2=4.6t$

p) $5x+4-4x=0$

r) $6x+5+7x+3=12x+4$

s) $2(p+5)-(9+p)=-3$

a) $4x-3(x+8)=5x-2(x-12)-2x$

t) $2(y+4)-2y=8$

u) $5x+8=2x+8$

v) $-3\left(x-\dfrac{1}{4}\right)=-4x$

x) $\dfrac{q}{2}+13=-22$

y) $3-3(5-t)=0$

z) $(3-3)(5-x)=0$

Exercise #13 Solve the following equations .

a) $\dfrac{3}{4}z-\dfrac{1}{4}=\dfrac{3}{4}$

b) $\dfrac{4}{5}y-\dfrac{1}{5}=\dfrac{3}{5}$

c) $\dfrac{x+4}{2}+\dfrac{x+1}{4}=3$

d) $\dfrac{w+3}{6}-\dfrac{w+4}{2}=2$

e) $\dfrac{2}{3}(v-4)=2$

f) $\dfrac{3}{4}(u-6)=2$

g) $\dfrac{5}{3}(t-1)=\dfrac{4}{5}(2t+1)+\dfrac{2}{3}$

h) $\dfrac{4}{5}(s+2)=\dfrac{1}{2}+\dfrac{5}{6}(s+3)$

i) $\dfrac{3(n-2)}{5}=\dfrac{3n+6}{6}$

j) $\dfrac{x}{3}+\dfrac{1}{6}=\dfrac{2}{5}$

k) $\dfrac{6}{7}m-\dfrac{3}{4}=\dfrac{4}{5}-\dfrac{1}{7}m+\dfrac{1}{6}$

l) $\dfrac{2}{3}k-\left(k-\dfrac{1}{2}\right)=\dfrac{1}{6}(k-51)$

m) $\frac{1}{2}(x-1)-\frac{3}{4}x+5=\frac{1}{6}$ n) $\frac{1}{3}(x+3)+\frac{1}{6}(x-6)=x+3$

o) $-\frac{5}{6}q-(q-1)=\frac{1}{4}(-q+80)$ p) $-\frac{1}{4}(x-12)+\frac{1}{2}(x+2)=x+4$

Exercise #14 Solve the following equations .

a) $0.8q-3.2=1.6$ b) $2.3r-4.7=4.5$ c) $2.3s+4.7s=4.9$

d) $5.1m+2.3m=2.96$ e) $0.4(0.2n-0.3)=0.01$ f) $0.8(0.3p-0.5)=0.8$

g) $0.8q-0.3(210-q)=80$ h) $0.3r+1.2(20)=0.8(r+20)$ i) $x+0.05(12-x)=0.1(63)$

j) $3(y-0.87)-2y=4.98$ k) $0.4y+0.3(20-y)=0.1y+6$ l) $0.1x+0.05(x-300)=105$

Exercise #15 Solve the following equations .

a) $15\%q=6$ b) $30\%r=9$ c) $50\%s+s=12$

d) $75\%t+t=105$ e) $20\%u+25\%u=18$ f) $50\%t+20\%(90-t)=30$

g) $20\%+40\%(25-s)=9$

Answers #13: a) 4/3; b) 1; c) 1; d) -21/2;e) 7; f) 26/3; g) 47; h) -42; i) 22 ; m) 52/3; o) -12
Answers #14: a) 6; b) 4; c) .7; d) .4; e) 13/8; f) 5; g) 130; h) 16
Answers #15: a) 40; b) 30; c) 8; d) 60; e) 40; f) 40; g) 3

Exercise #16

Evaluate $x^2-(xy-y)$ for x satisfying $\frac{3(x+3)}{5}=2x+6$ and y satisfying $-2y-10=5y+18$.

Exercise #17 Solve each formula for the specified variable:

a) $v=k+gt$, for t $\left(A:t=\dfrac{v-k}{g}\right)$

b) S=3**p**d+**p**a, for d $\left(A:d=\dfrac{S-pa}{3p}\right)$

c) $A=P(1+rt)$, for r $\left(A:r=\dfrac{A-p}{pt}\right)$

d) $A=2w^2+4lw$, for l $\left(A:l=\dfrac{A-2w^2}{4w}\right)$

e) $A=\frac{1}{2}h(a+b)$ for a $\left(A:a=\dfrac{2A}{h}-b\right)$

f) $A=2lw+2lh+2wh$ for l $\left(A:l=\dfrac{A-2wh}{2(w+h)}\right)$