Circles
Sections 6.1 - 6.4

The many practical uses of the circle range from the wheel to the near-circular orbits of some communication satellites. The mechanical uses of the circle have been known for thousands of years, and the ancient Greeks contributed significantly to our understanding of the circle's mathematical properties. The full moon, ripples in a pond when a stone is dropped in, and the shape of some bird's nests show some of the circles that appear in nature.

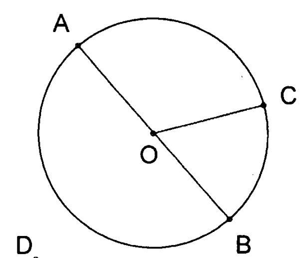
Our study of circles begins with some definitions, an explanation of the standard symbols used, and certain figures related to circles.

Definition (6.1) A **circle** is the set of all points in a plane that are at a given distance from a given point in the plane.

The given distance = $OA = OC = OB = r$

The given point = center O

Notation: $\odot O$ - the circle with center O



Note: A circle divides the plane into three distinct subsets:

- the interior $O \in \text{int} \odot O$
- the circle itself $A, B, C \in \odot O$
- the exterior $D \in \text{ext} \odot O$

Note: The radius of a circle is defined above as a number. It is standard practice, however, for "radius" to also mean a line segment, as in the following definition. You can usually determine which meaning of the word " radius" is intended by the context in which it is used.

Definition A **radius** of a circle is a segment that joins the center of the circle to a point on the circle. $\overline{OA}$

Definition (6.1) A **diameter** of a circle is a segment whose endpoints are points of the circle and it contains the center of the circle. $AB = 2r$ $\overline{AB}$

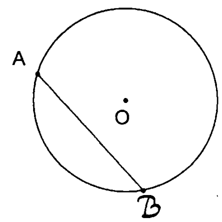
Theorem In any given circle all radii are congruent and all diameters are congruent. (radii $\odot \cong$ and diams $\odot \cong$)

Postulate (6.1 – P6.1) Two or more **circles are congruent** if and only if they have congruent radii ($\odot s \cong$ iff radii $\cong$).

Definition Two or more coplanar **circles are concentric** if they have the same center.

Question: How many circles can share the same center? infinitely many

Definition (6.2) A line segment is a **chord** of a circle if its endpoints are points of the circle. $\overline{AB}$ = chord



Questions:

1) Is a diameter a chord? Yes

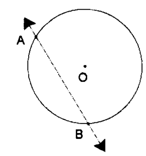
2) Is a radius of chord? No

3) What is the longest possible chord? the diameter

4) How is the length of a chord related to its distance from the center? the closer to the center, the longer the chord

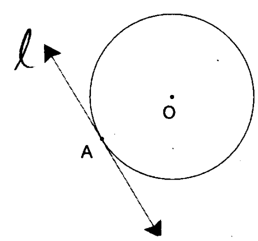
Definition (6.2) A line (or segment or ray) is a **secant** if it intersects a circle at exactly twp points.

$\overleftrightarrow{AB}$ - secant $\overleftrightarrow{AB} \cap \odot O = \{A, B\}$

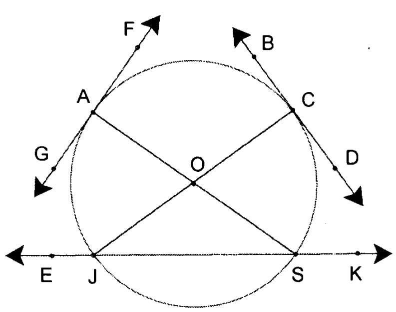


Definition (6.3) A line is a **tangent** to a circle if it intersects the circle at exactly one point.

l - tangent $l \cap \odot O = \{A\}$



Problem #1 In the given figure, name:



a) fours radii $\overline{OA}$, $\overline{OC}$, $\overline{OS}$, $\overline{OJ}$

b) two diameters $\overline{AS}$, $\overline{CJ}$

c) three chords $\overline{JS}$, $\overline{JC}$, $\overline{SA}$

d) two tangents $\overleftrightarrow{GF}$, $\overleftrightarrow{BD}$

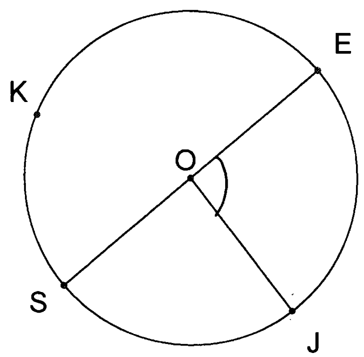
e) one secant $\overleftrightarrow{EK}$

Several types of angles associated with circles are seen in the above figure. The next definition describes the most fundamental of these angles.

Definition (6.1) An angle is a **central angle** of a circle if its vertex is the center of the circle

A central angle may be
- acute $\angle AOJ$
- right
- obtuse (measure less than 180°) $\angle JOS$

These angles "cut off" portions of the circle called arcs.



Definition (6.1) A **minor arc** is the set of points of a circle that are on a central angle or in its interior.

Example: $\overset{\frown}{EJ}$ (corresponding to $\angle EOJ$)

Definition The **intercepted arc of an angle** is the minor arc associated with the central angle.

Example: What is the intercepted arc of $\angle SOJ$? $\overset{\frown}{SJ}$

Definition (6.1) A **major arc** is the set of points of a circle that are on a central angle or in its exterior.

Example: $\overset{\frown}{EKJ}$ (corresponding to $\angle EOJ$)

Definition (6.1) A **semicircle** is the set of points of a circle that are on, or are on one side of, a line containing a diameter.

Example: $\overset{\frown}{SKE}$, $\overset{\frown}{SJE}$

Arc Addition Postulate (6.1)

Let A, B, and C be three points on the same circle with B between A and C. Then

$$m\overset{\frown}{AC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$

Definition (6.1)

The degree measure of

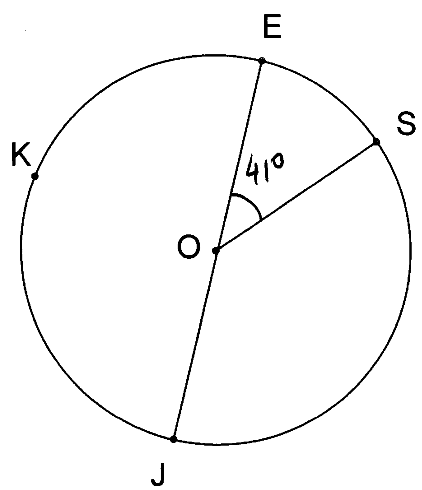
a) a minor arc is the measure of its central angle (also known as **The Central Angle Postulate**),

b) a semicircle is 180°,

c) a circle is 360°,

d) a major arc is 360° minus the measure of its associated minor arc.

Problem #2



Given: $\odot O$

$m\angle EOS = 41^\circ$

Find:

$m\overset{\frown}{ES}$ $m\overset{\frown}{ES} = m\angle EOS = 41^\circ$

$m\overset{\frown}{ESJ}$ $m\overset{\frown}{ESJ} = 180^\circ$ (semicircle)

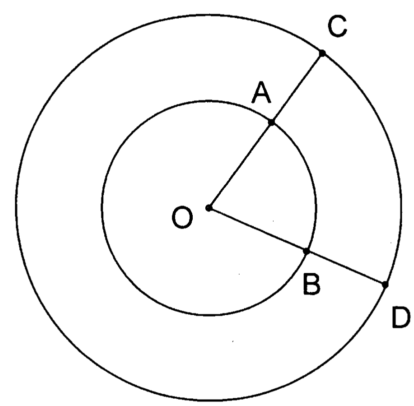
$m\angle SOJ$ $m\angle SOJ = 180^\circ - 41^\circ = 139^\circ$

$m\overset{\frown}{SJ}$ $m\overset{\frown}{SJ} = m\angle SOJ = 139^\circ$

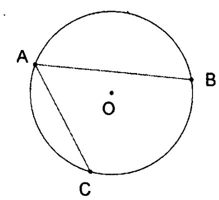
$m\overset{\frown}{EKS}$ $m\overset{\frown}{EKS} = 360^\circ - 41^\circ = 319^\circ$

Note: The degree measure of an arc is not a measure of the arc's length.

For the concentric circles in the figure,
$m\overset{\frown}{AB} = m\overset{\frown}{CD}$ because the arcs have the same central angle,
but certainly $\overset{\frown}{AB}$ is not as long as $\overset{\frown}{CD}$.



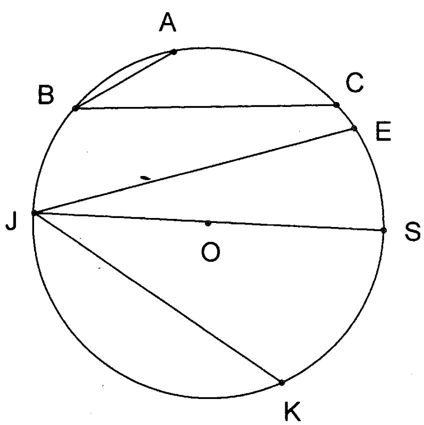
Inscribed Angles (6.1)



Definition (6.1) An angle is an **inscribed angle** of a circle if its vertex is a point on the circle and its sides are chords of the circle.

$< BAC$

There are three different types of inscribed angles when considered in relation to the center of the circle.



1) One side of the angle may contain a diameter, as do $< EJS, < SJK$

2) The circle's center may be in the angle's interior as is the case for $< EJK$

3) The center may be in the angle's exterior as it is for $< ABC$

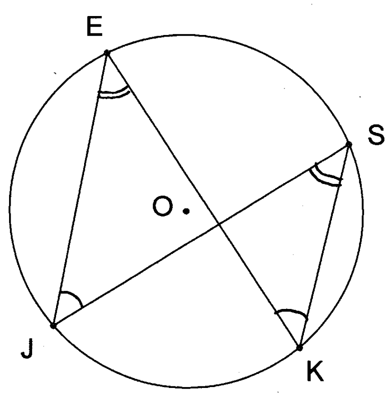
Theorem 1 (6.1 – T 6.2)

> The measure of an inscribed angle is equal to one-half the degree measure of its intercepted arc. (inscr $\angle = \frac{1}{2}\frown$)

Example: $m < ABC = \frac{1}{2} m \overset{\frown}{AC}$; $m < EJS = \frac{1}{2} m \overset{\frown}{ES}$; $m < EJK = \frac{1}{2} m \overset{\frown}{EK}$

Theorem 2 (6.1 – C 6.3)

> If two inscribed angles in a circle intercept the same arc or congruent arcs, then the angles are congruent (inscr $\angle s$ intercept same $\frown$ or $\cong \frown s$ are $\cong$).



$< JEK$ and $< JSK$ - intercept arc $\overset{\frown}{JK}$ $\Rightarrow$

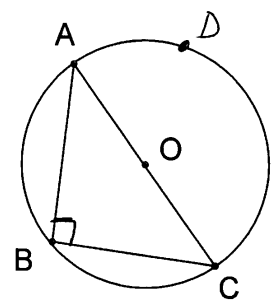
$< JEK \cong < JSK$

$< EJS$ and $< EKS$ - intercept arc $\overset{\frown}{ES}$ $\Rightarrow$

$< EJS \cong < EKS$

Theorem 3 (6.1 – C 6.4)

> If an inscribed angle intercepts a semicircle, then it is a right angle (inscr $\angle$ interc semi$\odot$ is rt$\angle$).



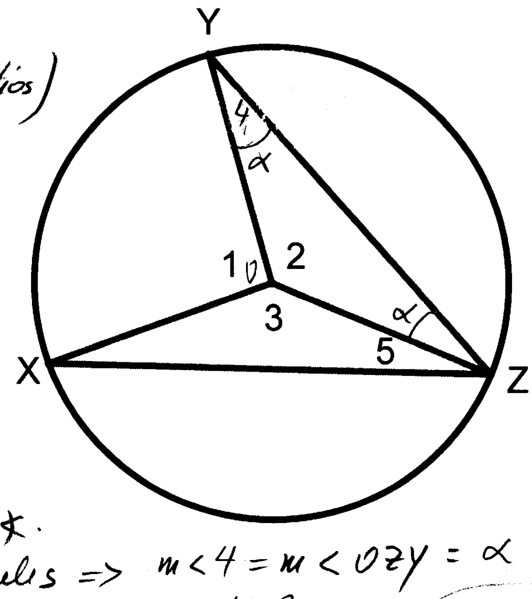
$< ABC$ = inscribed $\angle$

$\overline{AC}$ = diameter

$m < ABC = \frac{1}{2} m \overset{\frown}{CDA} = \frac{1}{2} 180° = 90°$

Problem #3 Use the figure to answer the questions.

(6.1 - # 21 – 24) $m\widehat{XY} : m\widehat{YZ} : m\widehat{ZX} = 5:6:7$ (extended ratios)



a) Find $m\widehat{XY}$, $m\widehat{YZ}$, and $m\widehat{ZX}$.

Let $m\widehat{XY} = 5x$

$m\widehat{YZ} = 6x$

$m\widehat{ZX} = 7x$

Then $5x + 6x + 7x = 360°$

$x = 20$

$m\widehat{XY} = 100°$

$m\widehat{YZ} = 120°$

$m\widehat{ZX} = 140°$

b) Find the measure of angles 1 to 5.

$\angle 1, \angle 2$, and $\angle 3$ are central ∡'s

$m\angle 1 = m\widehat{XY} = 100° = m\angle 1$

$m\angle 2 = m\widehat{YZ} = 120° = m\angle 2$

$m\angle 3 = m\widehat{XZ} = 140° = m\angle 3$

$\angle 4$ = inscribed ∡.

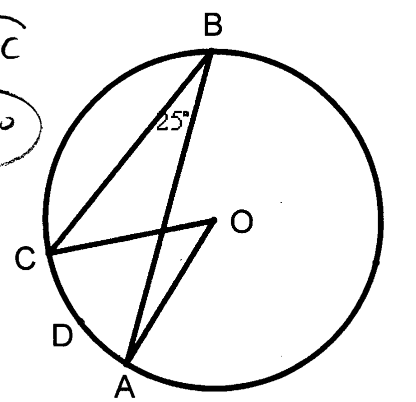
$\triangle YOZ$ - isosceles $\Rightarrow m\angle 4 = m\angle OZY = \alpha$

$\triangle YOZ: \alpha + \alpha + m\angle 2 = 180°$

$2\alpha + 120° = 180° \Rightarrow \alpha = m\angle 4 = 30°$

Problem #4 Use the figure to answer each question.

(6.1 - # 26)



a) What is $m\angle AOC$ - central ∡ $\Rightarrow m\angle AOC = m\widehat{AC}$

but $m\angle CBA = 25° = \frac{1}{2} m\widehat{AC} \Rightarrow m\widehat{AC} = 50°$

so $m\angle AOC = 50°$

b) What is $m\widehat{AC}$

$m\angle \widehat{AC} = 50°$ (see (a))

c) What is $m\widehat{ABC}$?

$m\widehat{ABC} = 360° - m\widehat{AC} = 360° - 50° = 310° = m\widehat{ABC}$

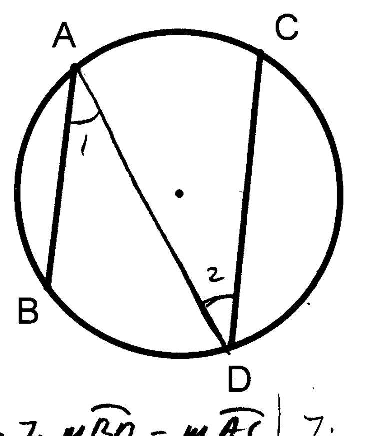
d) What is the measure of the arc intercepted by $\angle ABC$?

The arc intercepted by $\angle ABC$ is $\widehat{AC}$

$m\widehat{AC} = 50°$

Problem # 5 Given: $\overline{AB} \parallel \overline{CD}$

(6.1 - # 30) Prove: $m\widehat{AC} = m\widehat{BD}$



Proof

1. $\overline{AB} \parallel \overline{CD}$	1. given
2. Let $\overline{AD}$ - transv.	2. two points determine a line
3. $\angle 1 \cong \angle 2$	3. ∥ iff alt. int. ∡'s ≅ ($\overline{AB} \parallel \overline{CD}$ with transversal $\overline{AD}$)
4. $m\angle 1 = m\angle 2$	4. def ≅ angles
5. $m\angle 1 = \frac{1}{2} m\widehat{BD}$, $m\angle 2 = \frac{1}{2} m\widehat{AC}$	5 inscribed ∡ = $\frac{1}{2}$ intercepted arc
6. $\frac{1}{2} m\widehat{BD} = \frac{1}{2} m\widehat{AC}$	6. substitution (4,5)
7. $m\widehat{BD} = m\widehat{AC}$	7. ·/÷ prop of =

Central Angles, Arcs, and Chords
6.2

There are some important properties about central angles, arcs, and chords that are associated with a given circle or with two circles that are the same size. But what is meant by "the same size"? congruent circles

Definition Two **arcs** of a circle or of congruent circles are congruent iff their degree measures are equal.

Note: Since congruent arcs are defined in terms of numbers (degree measures), the addition, subtraction, multiplication, and division properties of congruence may be easily extended to include congruence between arcs.

Theorem 4
(6.2 – T 6.6)

If two chords in a circle or in congruent circles are congruent, then their arcs are congruent (if chords $\cong$, $\widehat{\ }s \cong$).

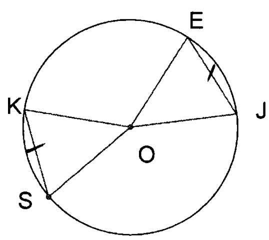
Write a formal proof.

Given $\odot O$
$\overline{EJ} \cong \overline{KS}$

Prove $\widehat{EJ} \cong \widehat{KS}$

Proof

see textbook



Theorem 5 (Converse of Theorem 4)
(6.2 – T. 6.7)

if two arcs in a circle or in congruent circles are congruent, then their corresponding chords are congruent
(if $\widehat{\ }s \cong$, chords $\cong$).

Given $\odot O$
$\widehat{EJ} \cong \widehat{KS}$

Prove $\overline{EJ} \cong \overline{KS}$

Remember that the measure of a central angle is equal to the measure of its *intercepted arc*

Therefore, we have the following property (theorem):

Theorem 6

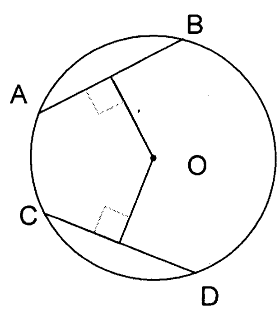
> Two minor arcs of a circle or of congruent circles are congruent if and only if their central angles are congruent ($\overset{\frown}{}s \cong$ *iff central* $\angle s \cong$).

The above three theorems are summarized in the following diagram:

> $\cong$ central angles $\leftrightarrow$ $\cong$ arcs $\leftrightarrow$ $\cong$ chords.

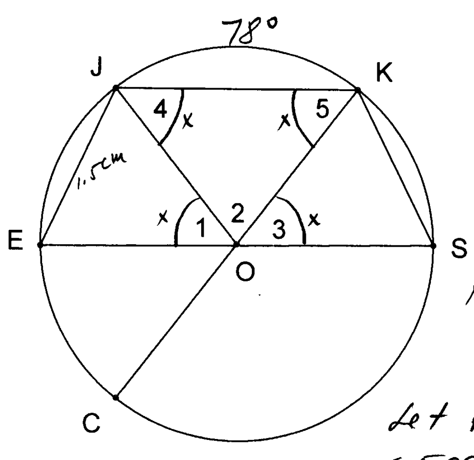
Theorem 7
(6.2 – T. 6.10 and T. 6.11)

> Chords are at the same distance from the center of a circle if and only if they are congruent.



$dist(O, \overline{AB}) = dist(O, \overline{CD})$ iff $\overline{AB} \cong \overline{CD}$

Problem #6



Given: $\odot O$
$\overline{ES} \parallel \overline{JK}$
$m\overset{\frown}{JK} = 78^\circ$
$JE = 1.5$ cm

Find:
a) $\angle s1-5$
b) $m\overset{\frown}{JE}$, $m\overset{\frown}{KS}$, KS, $m\overset{\frown}{JC}$

Solution

Note that $\angle 1 \cong \angle 4 \cong \angle 5 \cong \angle 3$
(alt. int $\angle$'s) (isosc. Δ) (alt. int)

$m\angle 2 = m\overset{\frown}{JK} = 78^\circ$

Let $m\angle 1 = x$

$\angle EOS$ = straight $\angle$ $\Rightarrow$ $x + m\angle 2 + x = 180^\circ \Rightarrow x = 51^\circ$

Therefore,
(a)
$m\angle 1 = 51^\circ$
$m\angle 2 = 78^\circ$
$m\angle 3 = 51^\circ$
$m\angle 4 = 51^\circ$
$m\angle 5 = 51^\circ$

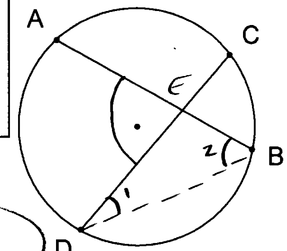
(b) $m\overset{\frown}{JE} = m\angle 1 = 51^\circ$
$m\overset{\frown}{KS} = m\angle 3 = 51^\circ$
$m\overset{\frown}{JC} = 180^\circ - m\overset{\frown}{JK}$
$= 102^\circ$

$KS = JE = 1.5$ cm
($\cong$ central $\angle$'s iff $\cong$ chords)

Chords, Tangents, and Secants
6.2, 6.3

Theorem 8
(6.2 – T 6.5)

> The measure of an angle formed by two chords that intersect within a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
>
> (2 chords $\angle = \frac{1}{2}$ sum $\frown$'s).



Proof

$\angle AED$ - ext. $\angle$ for $\triangle EDB$

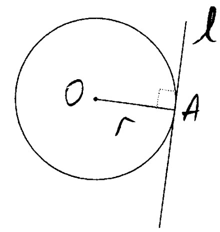
$m\angle AED = m\angle 1 + m\angle 2$

$= \frac{1}{2} m\overset{\frown}{CB} + \frac{1}{2} m\overset{\frown}{AD}$

$m\angle AED = \frac{1}{2}(m\overset{\frown}{CB} + m\overset{\frown}{AD})$

Postulate
(6.3 – P 6.3&4)

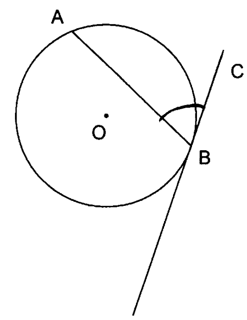
> A line is tangent to a circle, if and only if it is perpendicular to the radius drawn to the point of contact (tan $\perp$ rad to point contact).



l - tangent
$l \cap \odot O = \{A\}$ $\iff$ $\overline{OA} \perp l$

Theorem 9
(6.3 – T 6.16)

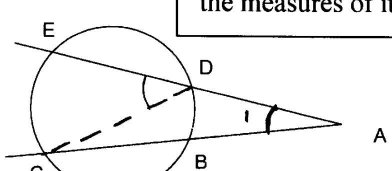
> The measure of an angle formed by a tangent to a circle and a chord drawn to the point of tangency is on-half the measure of its intercepted arc.

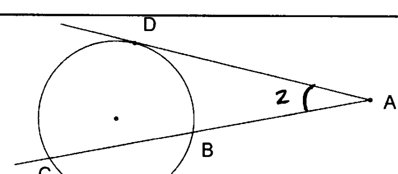


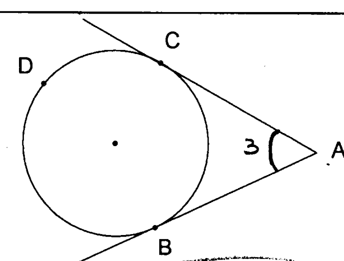
$m\angle ABC = \frac{1}{2} m\overset{\frown}{AB}$

Theorem 10
(6.2 – T 6.14,
6.3 – T 6.17 &18)

> If an angle is formed by
>
> - two secants ($\angle 1$)
>
> or
>
> - a tangent and a secant ($\angle 2$)
>
> or
>
> - two tangents ($\angle 3$)
>
> intersecting in the exterior of the circle, then the measure of the angle is one-half the difference of the measures of its intercepted arcs.







$\angle EDC$ - ext. $\angle$ $\triangle DCA$

$m\angle EDC = m\angle DCA + m\angle 1$

$m\angle 1 = m\angle EDC - m\angle DCA$

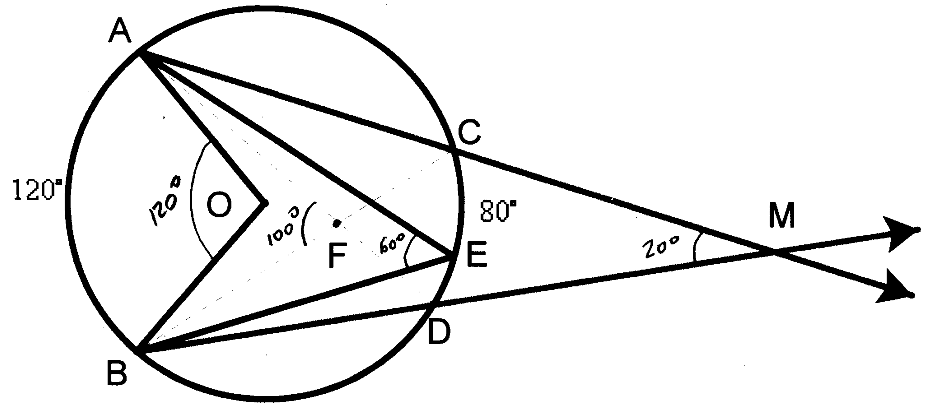
$= \frac{1}{2} m\overset{\frown}{EC} - \frac{1}{2} m\overset{\frown}{DB}$

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{EC} - m\overset{\frown}{DB})$

$m\angle 2 = \frac{1}{2} m(\overset{\frown}{DC} - \overset{\frown}{BD})$

$m\angle 3 = \frac{1}{2}(m\overset{\frown}{CDB} - m\overset{\frown}{CB})$

Problem #7 The next figure suggests a way to remember some of the properties of angles and arcs in circles. Note that the sizes of the angles decrease from left to right and that O is the circle's center. The following arcs and angles are shown in the figure:



Given arcs: $m\overset{\frown}{AB} = 120^\circ$ and $m\overset{\frown}{CD} = 80^\circ$

Central angle: $m\angle AOB = m\overset{\frown}{AB} = 120^\circ$

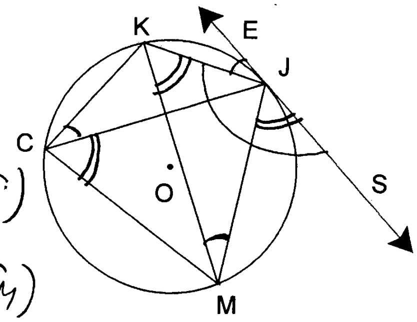
Angle formed by 2 chords: $m\angle AFB = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$
$= \frac{1}{2}(120^\circ + 80^\circ) = 100^\circ$

Inscribed angle: $m\angle AEB = \frac{1}{2}m\overset{\frown}{AB} = 60^\circ$

Angle formed by two secants:

$$m\angle AMB = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})$$
$$= \frac{1}{2}(120^\circ - 80^\circ)$$
$$= 20^\circ$$

Problem #8 Use the figure to answer the questions.



Given $\odot O$

tan $\overrightarrow{ES}$

a) Name two angles congruent to $\angle KJE$.

$\angle KMJ$ and $\angle KCJ$ (intercepted arc $\widehat{KJ}$)

b) Name two angles congruent to $\angle JCM$.

$\angle MKJ$ and $\angle MJS$ (intercepted arc $\widehat{JM}$)

c) Name three angles supplementary to $\angle KJS$.

$\angle EJK$, $\angle KMJ$, $\angle KCJ$ (b/c part (a))

d) Name one angle supplementary to $\angle KCM$.

$\angle KJM$

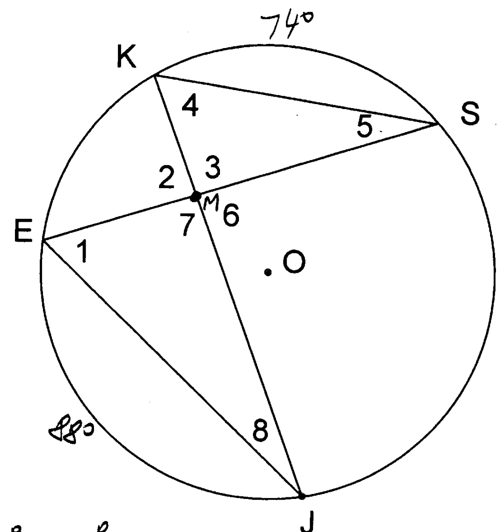
Problem #9 Given $\odot O$

$m\widehat{EJ} = 88^\circ$

$m\widehat{KS} = 74^\circ$

$m\angle 8 = \frac{1}{3} m\angle 2$

Find $m\angle 1 - 8$



Solution

- $m\angle 7 = m\angle 3 = \frac{1}{2}(m\widehat{KS} + m\widehat{EJ}) = \frac{1}{2}(74^\circ + 88^\circ) = 81^\circ$
  (vertical ∡'s)
- $m\angle 2 = m\angle 6 = \frac{1}{2}(m\widehat{KE} + m\widehat{SJ})$
  (vertical ∡'s)
  but $m\widehat{KE} + m\widehat{SJ} = 360^\circ - (m\widehat{KS} + m\widehat{EJ}) = 360^\circ - 162^\circ = 198^\circ$
  so, $m\angle 2 = m\angle 6 = \frac{1}{2}(198^\circ) = 99^\circ$
- $m\angle 8 = \frac{1}{3} m\angle 2 = \frac{1}{3} 99^\circ = 33^\circ$
  (given)
- $\angle 2$ - exterior ∡ $\triangle EMJ \Rightarrow$
  $m\angle 2 = m\angle 1 + m\angle 8 \Rightarrow$
  $m\angle 1 = m\angle 2 - m\angle 8 = 99^\circ - 33^\circ = 66^\circ$
- $m\angle 4 = m\angle 1 = 66^\circ$ (inscribed ∡'s intercept same ⌒ are ≅)
- $m\angle 5 = m\angle 8 = 33^\circ$ (inscribed ∡'s intercept same ⌒ are ≅)

Check: $\triangle EMJ$: $m\angle 1 + m\angle 7 + m\angle 8 = 180^\circ$

$\triangle KMS$: $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$

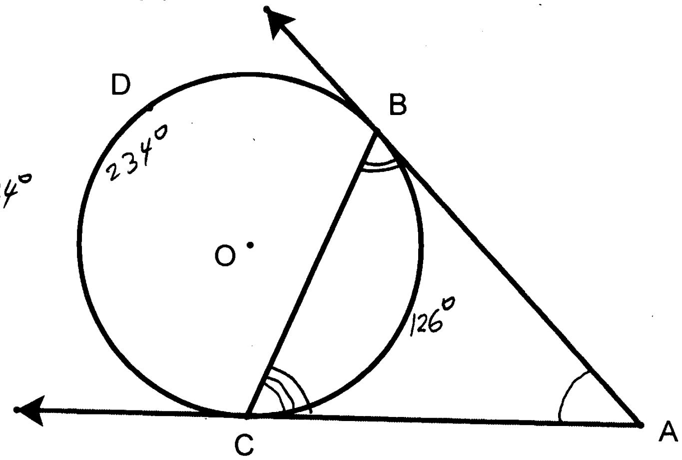
$m\angle 1 = 66^\circ$
$m\angle 2 = 99^\circ$
$m\angle 3 = 81^\circ$
$m\angle 4 = 66^\circ$
$m\angle 5 = 33^\circ$
$m\angle 6 = 99^\circ$
$m\angle 7 = 81^\circ$
$m\angle 8 = 33^\circ$

<u>Problem #10</u> Given: $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents to $\odot O$, $m\overset{\frown}{BC} = 126^\circ$.

Find: a) $m\angle A$

b) $m\angle ABC$

c) $m\angle ACB$



Solution

if $m\overset{\frown}{BC} = 126^\circ$, then $m\overset{\frown}{BDC} = 360^\circ - 126^\circ = 234^\circ$

$m\angle A = \frac{1}{2}(m\overset{\frown}{BDC} - m\overset{\frown}{BC})$

$= \frac{1}{2}(234^\circ - 126^\circ) = 54^\circ$

$m\angle ABC = \frac{1}{2} m\overset{\frown}{BC} = \frac{1}{2}(126^\circ) = 63^\circ$

$m\angle ACB = \frac{1}{2} m\overset{\frown}{BC} = 63^\circ$

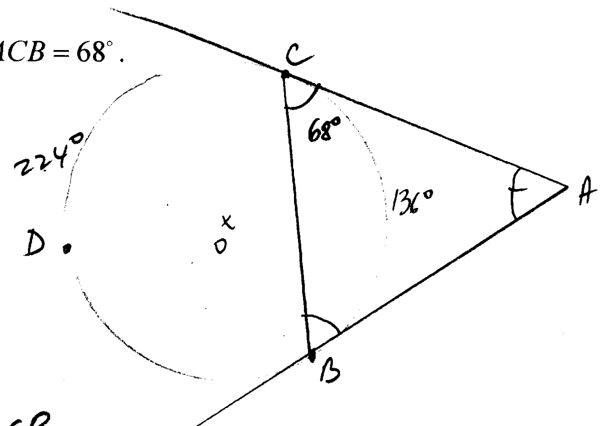
<u>Problem #11</u> Given: $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents to $\odot O$, $m\angle ACB = 68^\circ$.

Find: a) $m\overset{\frown}{BC}$ $B, C \in \odot O$

b) $m\overset{\frown}{BDC}$, $D \in \odot O$

c) $m\angle ABC$

d) $m\angle A$



Solution

(a) $m\angle ACB = 68^\circ$. given

$m\angle ACB = \frac{1}{2} m\overset{\frown}{BC} \Rightarrow m\overset{\frown}{BC} = 2m\angle ACB$

$= 2 \cdot 68^\circ = 136^\circ$

(b) $m\overset{\frown}{BDC} = 360^\circ - m\overset{\frown}{BC}$

$= 360^\circ - 136^\circ = 224^\circ$

(c) $m\angle ABC = m\angle ACB = 68^\circ$ (intercept same arc $\overset{\frown}{BC}$)

(d) $m\angle A = \frac{1}{2}(m\overset{\frown}{CDB} - m\overset{\frown}{BC})$

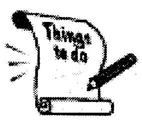
$= \frac{1}{2}(224^\circ - 136^\circ) = 44^\circ$

(or use $\triangle ABC$, sum $\angle$'s 180°)

Line and Segment Relationships in the Circle
Lengths of Segments in a Circle
6.2, 6.3

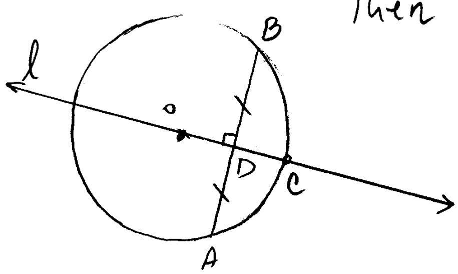
Theorem 11
(6.2 – T 6.8)

A line drawn from the center of a circle perpendicular to a chord bisects the chord and the arc formed by the chord (sec thru center ⊥ chord bisects chord & arc).



$\odot O$
$\overline{AB}$ - chord
l - line, $O \in l$, $l \cap \overline{AB} = \{D\}$, $l \cap \widehat{AB} = \{C\}$

Then $\overleftrightarrow{OD} \perp \overline{AB}$ iff $\overline{AD} \cong \overline{BD}$
$\widehat{BC} \cong \widehat{AC}$



See textbook page 287.

Theorem 12 (Converse of Theorem 8)
(6.2 – T 6.9)

see textbook

Theorem 13
(6.2 – T6.12)

The perpendicular bisector of a chord passes through the center of the circle.

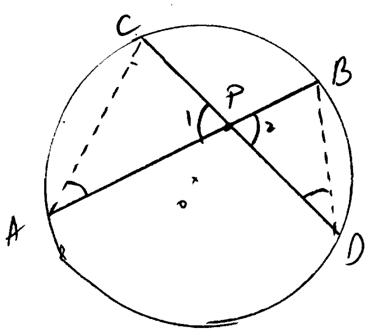
Theorem 14
(6.3 – T 6.19)

The tangent segments to a circle from an external point are congruent (tans to ⊙ ≅).

see textbook

Theorem 15
(6.2 – T 6.13)

If two chords intersect inside a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other.



Given ⊙
$\overline{AB}, \overline{CD}$ - chords
$\overline{AB} \cap \overline{CD} = \{P\}$

Prove $\boxed{AP \cdot PB = CP \cdot PD}$

(comes from ~ Δ's)

Proof

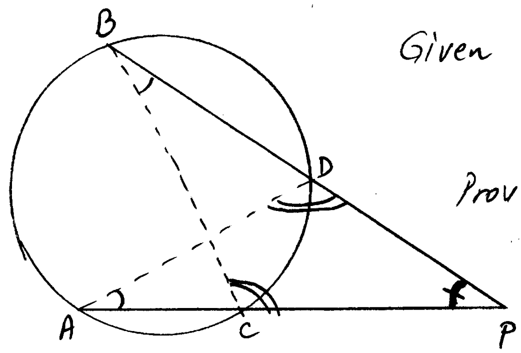
$\triangle APC$, $\triangle DPB$: $<1 \cong <2$ (vertical); $<A \cong <D$ (intercept same arc $\overset{\frown}{BC}$)

$\Rightarrow \triangle APC \sim \triangle DPB$ (AA)

$\Rightarrow \frac{AP}{PD} = \frac{CP}{PB} \Rightarrow AP \cdot PB = CP \cdot PD$

Theorem 16
(6.2 – T 6.15)

If two secants are drawn to a circle from an external point, then the product of the lengths of one secant segment to its external segment is equal to the product of the lengths of the other secant segment and its external segment.



Given ⊙
$\overline{PA}, \overline{PB}$ - secants

Prove $\boxed{PA \cdot PC = PB \cdot PD}$

Proof

$\triangle PBC$, $\triangle PAD$: $<P \cong <P$ (common ∢); $<B \cong <A$ (intercept same arc $\overset{\frown}{CD}$)

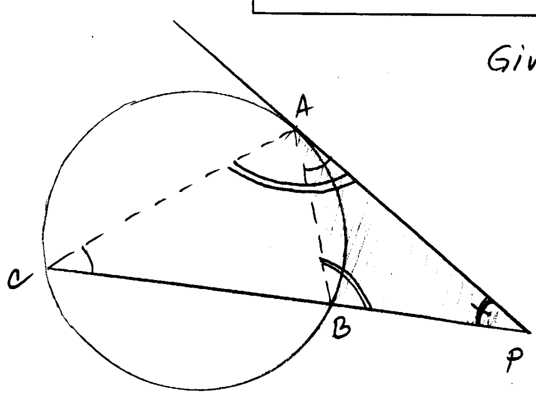
$\Rightarrow \triangle PBC \sim \triangle PAD$ (AA) $\Rightarrow$

$\frac{PB}{PA} = \frac{PC}{PD} \Rightarrow$

$PA \cdot PC = PB \cdot PD$

Theorem 17
(6.3 – T 6.20)

If a secant and a tangent are drawn to a circle from an external point, then the length of the tangent segment is the geometric mean between the length of the secant segment and its external segment.



Given ⊙
$\overrightarrow{PA}$ - tangent
$\overline{PC}$ - secant

Prove $\boxed{PA^2 = PC \cdot PB}$

Proof

$\triangle PAB$, $\triangle PCA$: $<P \cong <P$ (common ∢); $<PAB \cong <PCA$ (intercept same arc $\overset{\frown}{AB}$)

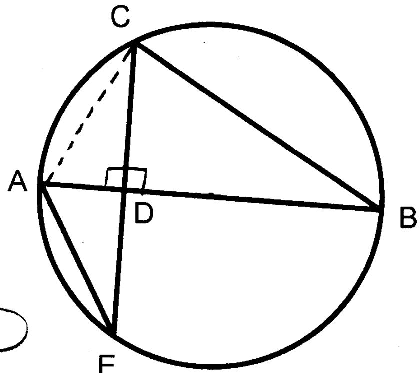
$\Rightarrow \triangle PAB \sim \triangle PCA$ (AA) $\Rightarrow$

$\frac{PA}{PC} = \frac{PB}{PA} \Rightarrow$

$PA^2 = PB \cdot PC$

Problem #12 Given: Diameter $\overline{AB} \perp \overline{CE}$ at D
Prove: CD is the geometric mean of AD and DB.

Condition: $CD^2 = AD \cdot DB$



Solution

Method I

$\triangle ACB$: $\overline{CD}$ = altitude

$CD^2 = AD \cdot DB$

(alt = geom. mean segm. hyp)

Method II

$\overline{AB}, \overline{CE}$ - secants $\Rightarrow$

$AD \cdot DB = CD \cdot DE$

$\overline{AB} \perp \overline{CE} \Rightarrow CD = DE$

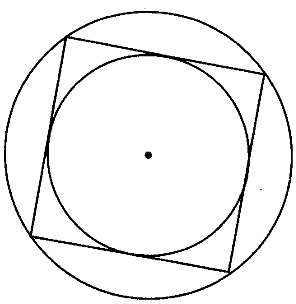
(secant $\perp$ chord bisects chord) $\Rightarrow$

$AD \cdot DB = CD^2$

Polygons inscribed in a circle
6.4

Definition Any **polygon is inscribed in a circle** if and only if all its vertices are points of the circle; the **circle is** said to be **circumscribed about the polygon**.

Also, a **circle is inscribed in a polygon** if and only if it is tangent to each of the polygon's sides.

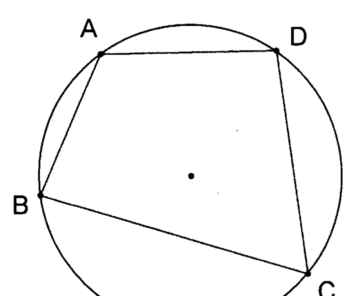


Example:

- the square is inscribed in the larger circle
- the larger circle is circumscribed about the square.
- the Smaller circle is inscribed in the square.

Theorem 18 (6.4 – T 6.23)

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary (if quad inscr in $\odot$, opp $\angle$ s supp).



$\angle B$ and $\angle D$ are supplementary because

$m\angle B + m\angle D = \frac{1}{2} m\overset{\frown}{ADC} + \frac{1}{2} m\overset{\frown}{ABC}$

$= \frac{1}{2}(m\overset{\frown}{ADC} + m\overset{\frown}{ABC})$

$= \frac{1}{2}(360°) = 180°$.