Postulates

Geometry, or any deductive system, is very much like a game. Before playing the game, it is necessary to accept some basic rules, which we will call *postulates*. The postulates in geometry are man-made, just as the rules of football are, and what the subject will be like depends upon the nature of the postulates used. We will study the geometry called Euclidean, named after Euclid. For many centuries, it was the only geometry known, because it took man a long time to realize that more than one set of rules were possible.

Geometry has very few rules. We will need to supplement them with some of the rules of algebra with which you are already familiar. The rules, or postulates, of algebra concern numbers and operations performed on them.

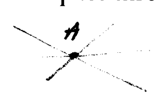
<u>Properties of Equality</u> (1.2 – Postulates 1.6 – 1.12)

Reflexive Property	Any real number is equal to itself. $a = a$
Symmetric Property	If $a = b$, then $b = a$
Transitive Property	If $a = b$ and $b = c$, then $a = c$
Addition Property	If $a = b$, then $a + c = b + c$ $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$ $\frac{a}{c} = \frac{b}{c}, \forall c \neq 0$.
Distributive Property	$a(b + c) = ab + ac$
Substitution Property	If $a = b$, then *a* can be substituted for *b* in any expression containing *b*.

The postulates of geometry deal with sets of points and their relationships.

<u>Question</u> Consider a single point. How many lines can pass through, or contain, it?

an infinite number

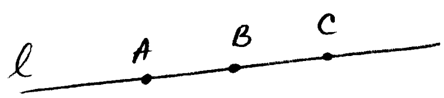


<u>Question</u> Now consider two points. How many lines can contain them?

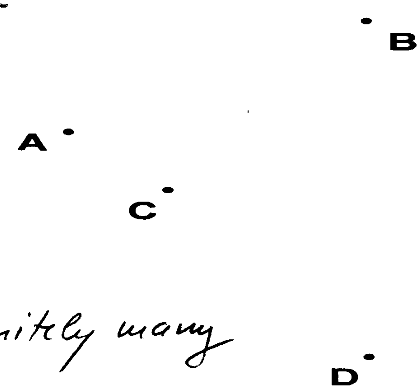
One (we think of a line as being straight and as having no length)

<u>Postulate 1:</u> Through two distinct points, there is exactly one line. (Two points determine a line.)

Definition Points that lie on the same line are called **collinear points.**



$A, B, C \in \ell$



Exercise #1

a) Name three points that appear to be collinear.

A, C, D

b) Name three points that appear to be noncollinear.

A, C, B

c) How many lines can be drawn through point *A*? infinitely many

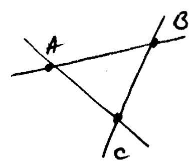
d) How many lines can be drawn through points A and *B*?

one

e) How many lines can be drawn through points *A*, *B*, and *C*? none

Exercise #2

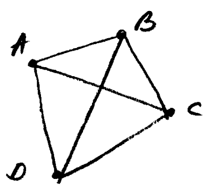
a) Make a drawing to illustrate three noncollinear points A, B, and C, and all of the lines they determine. How many lines are there in all?



$\overleftrightarrow{AB}$
$\overleftrightarrow{AC}$
$\overleftrightarrow{BC}$.

3 lines

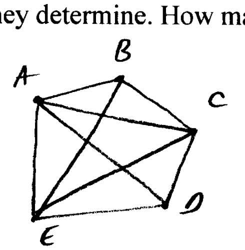
b) Make a drawing to illustrate four points, no three of which are collinear, and all of the lines they determine. How many lines are there in all?



$\overleftrightarrow{AB}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CD}$
$\overleftrightarrow{AC}$ $\overleftrightarrow{BD}$
$\overleftrightarrow{AD}$

$3+2+1 = 6$ lines

c) Make a drawing to illustrate five points, no three of which are collinear, and all of the lines they determine. How many lines are there in all?



$\overleftrightarrow{AB}$ $\overleftrightarrow{BC}$ $\overleftrightarrow{CD}$ $\overleftrightarrow{DE}$
$\overleftrightarrow{AC}$ $\overleftrightarrow{BD}$ $\overleftrightarrow{CE}$
$\overleftrightarrow{AD}$ $\overleftrightarrow{BE}$
$\overleftrightarrow{AE}$

$4+3+2+1 = 10$ lines

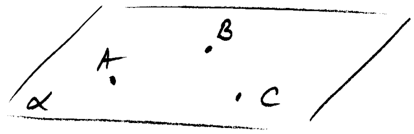
d) Without making a drawing, can you figure out how many lines are determined by ten points, no three of which are collinear?

$9+8+7+6+5+4+3+2+1$

lines

Postulate 2

> Through three noncollinear points, there is exactly one plane.
> (Three noncollinear points determine a plane).

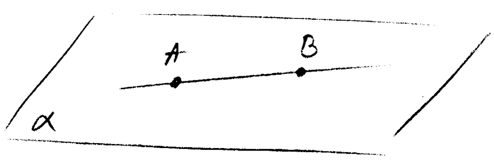


$A, B, C \in \alpha$

Definition Points that lie in the same plane are called **coplanar points.**

Postulate 3

> Given two distinct points in a plane, the line containing these points also lies in the plane.



if $A, B \in \alpha$,
then $\overleftrightarrow{AB} \subset \alpha$

Postulate 4

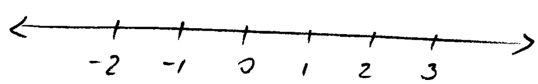
> No plane contains all points in space.

Space contains at least four points that are not all in the same plane

Postulate 5

> There is a one-to-one correspondence between the set of all points on a line and the set of all real numbers.

Number line:



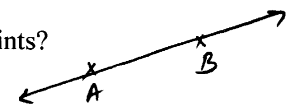
- 0 - origin
- The number corresponding to a given point on the line is called the coordinate of the point.
- When we identify a point with a given real number, we are plotting the point associated with the number.

Exercise #3
(1.2 - #1,3)

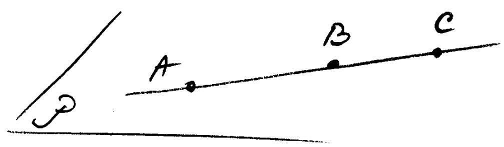
Answer each question and make a drawing to illustrate each situation.

1. How many lines can be drawn between two distinct points?

One , $\overleftrightarrow{AB}$



2. If distinct points A and B are in a plane $\mathcal{P}$, and point C is on the line determined by A and B, what can be said about point C?



if $A, B \in \alpha$
$C \in \overleftrightarrow{AB}$
then $C \in \alpha$

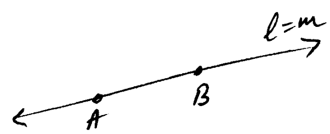
Exercise #4
(1.2 - # 5, 6)

Identity the hypothesis and conclusion of each statement.
Name the postulate illustrated by each statement.
Make a drawing to illustrate each situation.
A, B, and C are distinct points; l is a line; α is a plane.

1. If A and B are on l, and A and B are on m, then $m = l$.

Hypothesis: $\begin{cases} A, B \in l \\ A, B \in m \end{cases}$

Conclusion: $m = l$

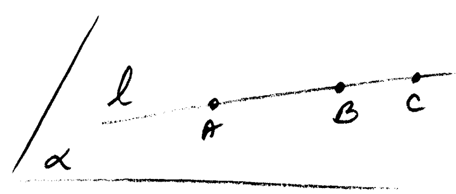


Postulate
Two points determine a line

2. If A and B are on l, l is in α, and C is on l, then C is in α.

Hypothesis: $\begin{cases} A, B \in l \\ l \subset \alpha \\ C \in l \end{cases}$

Conclusion: $C \in \alpha$



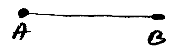
Postulate
Given two points in a plane, the line determined by the points is also in the plane

Exercise #5
(1.2 - #23)

Give the reasons that support each indicated step in the solution of the equation $3x + 2 = 4 + 5x$

Statements	Reasons
1. $3x+2=4+5x$	1. given
2. $3x+2-4=4-4+5x$	2. Addition prop. of equality
3. $3x-2=5x$	3. Simplify (combining like terms)
4. $3x-3x-2=5x-3x$	4. Addition prop. of equality
5. $-2=2x$	5. Simplify (combining like terms)
6. $\frac{1}{2}(-2)=\frac{1}{2}(2x)$	6. Multiplication prop. of equality
7. $-1=x$	7. Simplify
8. $x=-1$	8. Symmetric prop of equality

Definition A **line segment** is the part of a line that consists of two points (endpoints) and all points between them.



$\overline{AB}$

Question Is the above definition a good definition?

1. It names the term being defined: a line segment
2. It places the term into a set or category: part of a line
3. It distinguishes the defined term
4. It's reversible: The part of a line between and including 2 points is a line segment.

Exercise #6 a) You have learned that the following statement is true:

If a statement is a definition, then its converse is true.

Does it necessarily follow that if its converse is not true, a statement cannot be a definition? Explain. $(p \to q) \equiv (\sim q \to \sim p)$

Yes.

Decide which of the following true statements are good definitions of the italicized words by determining whether their converses are true.

b) If something is *cold*, then it has a low temperature. Good

If something has low temperature, it is cold.

c) A *mandolin* is a stringed musical instrument. Bad

A stringed musical instrument is not necessarily a mandolin

d) A *kitten* is a young cat. Good

A young cat is a kitten.

e) An *isosceles triangle* is a triangle that has two congruent sides. Good

If a triangle has two sides that are congruent, it's an isosceles triangle.

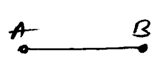
Note: When both a statement and its converse are true, there is a convenient way to combine the two into one. It is by means of the phrase ***"if and only if"***.

Statement: If P, then Q $\quad P \to Q \quad$ true

its converse: If Q, then P $\quad Q \to P \quad$ true

$\boxed{P \text{ if and only if } Q \quad P \leftrightarrow Q}$ $\left\{ \begin{array}{l} P \to Q \\ \text{and} \\ Q \to P \end{array} \right.$

Definition The **distance** between two points is the length of the line segment $\overline{AB}$ that joins the two points.



$\overline{AB}$ = segment AB

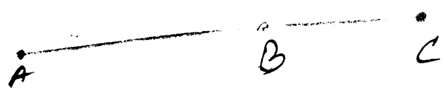
AB = length of the segment $\overline{AB}$

Example: Draw two points and find the distance between them.

Postulate 6:

> **Segment – Addition Postulate**
> If B is a point of $\overline{AB}$ and $A - B - C$, then $AB + BC = AC$

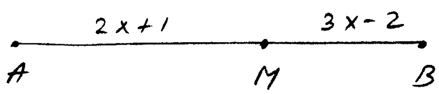
A-B-C means B is between A and



$AB + BC = AC$

Exercise #7

Given: M a point on $\overline{AB}$
$AM = 2x + 1$
$MB = 3x - 2$
$AB = 4$
Find: x, AM, and MB.

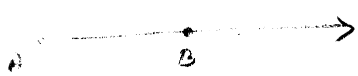


Solution

Statements	Reasons
1. $M \in AB$	1. given
2. $AM + MB = AB$	2. Segment-Addition Postulate
3. $AM = 2x+1$, $MB = 3x-2$, $AB = 4$	3. given
4. $(2x+1)+(3x-2) = 4$	4. Substitution
(2,3) 5. $5x - 1 = 4$	5. Simplifying (Combining like terms)
6. $5x = 5$	6. Addition property of equality
7. $x = 1$	7. Division property of equality
8. $AM = 2(1)+1 = 3$; $MB = 3(1)-2 = 1$	8. substitution

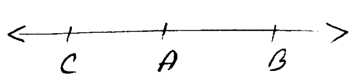
(7,3)

Definition **Ray** AB, denoted by $\overrightarrow{AB}$, consists of the point A (endpoint) together with all the points on the line $\overleftrightarrow{AB}$ on the same side of A as B.



$\overrightarrow{AB}$

Definition Two rays are **opposite rays** if they have a common endpoint and if their union is a straight line.



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays

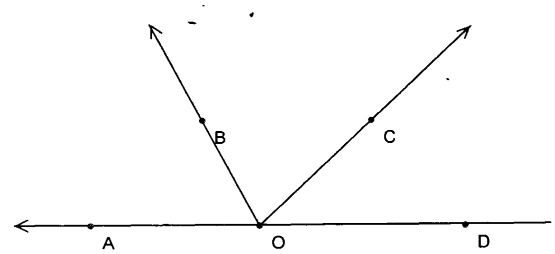
Exercise #8

In the figure, name:

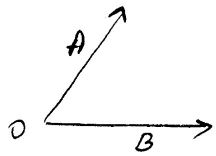
a) two opposite rays. $\overrightarrow{OA}$, $\overrightarrow{OD}$

b) two rays that are not opposite.

$\overrightarrow{OA}$ and $\overrightarrow{OC}$



Definition An **angle** is the union of two rays that share a common endpoint.



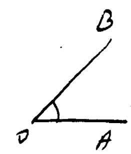
$\angle AOB$

Example Draw an angle, name it, and measure it.

Types of Angles

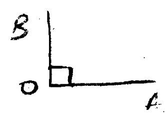
ACUTE ANGLE – an angle whose measure is less than 90°.

$m\measuredangle(AOB) < 90^\circ$



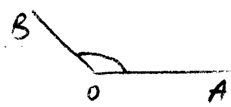
RIGHT ANGLE – an angle whose measure is exactly 90°.

$m\measuredangle AOB = 90^\circ$



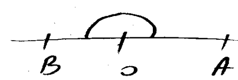
OBTUSE ANGLE – an angle whose measure is between 90° and 180°.

$m\measuredangle AOB > 90^\circ$



STRAIGHT ANGLE – an angle whose measure is exactly 180°.

$m\measuredangle AOB = 180^\circ$



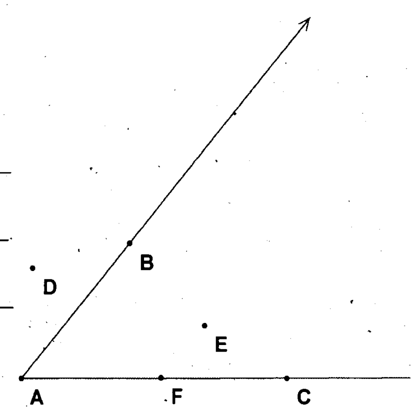
Exercise #9

Given the figure, which points lie

in the **interior** of $\angle BAC$? $E \in \text{int} \angle BAC$

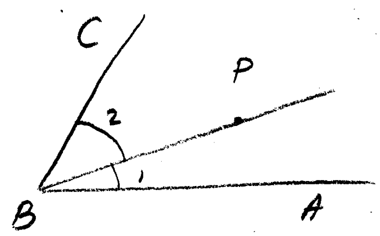
on $\angle BAC$? B, F, C

in the exterior of $\angle BAC$? D



Postulate 7

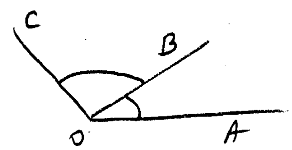
Angle – Addition Postulate
If a point P lies in the interior of an angle ABC, then
$m\angle ABP + m\angle PBC = m\angle ABC$



$m\measuredangle 1 + m\measuredangle 2 = m\measuredangle B$

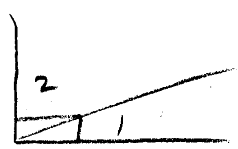
Classifying Pairs of Angles

Two angles are **adjacent angles** if they have a common vertex , share a common side, and have no interior points in common.



$\angle AOB$ and $\angle BOC$ are adjacent angles

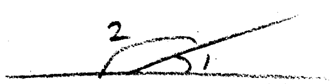
Two angles are **complementary** if their sum is 90°.



$m\measuredangle 1 + m\measuredangle 2 = 90^\circ$

$\measuredangle 1$ and $\measuredangle 2$ are complementary.

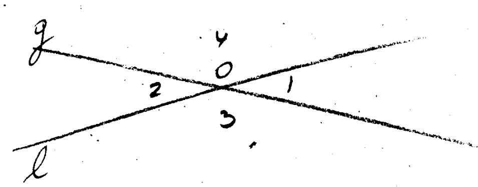
Two angle are **supplementary** if their sum is 180°.



$m\measuredangle 1 + m\measuredangle 2 = 180^\circ$

$\measuredangle 1$ and $\measuredangle 2$ are supplementary

When two lines intersect, the pairs of nonadjacent angles formed are known as **vertical angles.**



$\ell \cap g = \{O\}$

$\measuredangle 1$ and $\measuredangle 2$ are vertical angles

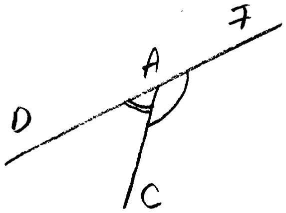
$\measuredangle 3$ and $\measuredangle 4$ are vertical angles.

Example Draw two intersecting lines.

a) Which angles are vertical angles?

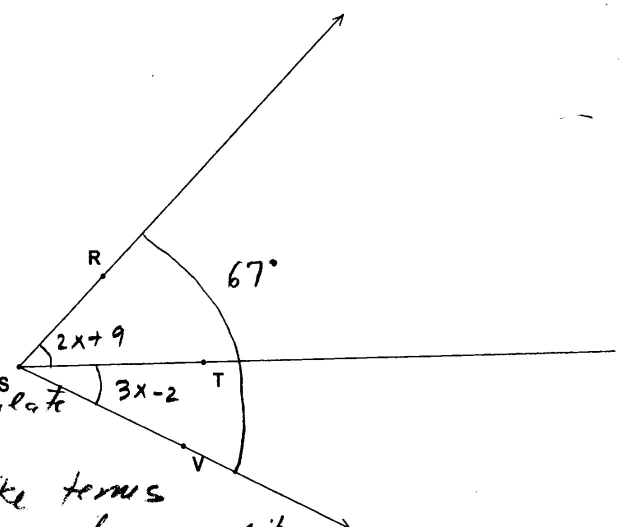
b) Which angles are supplementary?

<u>Exercise #10</u> $\angle FAC$ and $\angle CAD$ are adjacent and $\overrightarrow{AF}$ and $\overrightarrow{AD}$ are opposite rays. What can you conclude about $\angle FAC$ and $\angle CAD$?



∡ FAC and ∡ CAD are supplementary

<u>Exercise #11</u> Given: $m\angle RST = 2x + 9$
$m\angle TSV = 3x - 2$
$m\angle RSV = 67^\circ$
Find: x.



Solution

Statements	Reasons
1. T ∈ int ∡ RSV	1. given (figure)
2. m<TSV = 3x-2, m<RSV = 67	2. given
3 m∡RST + m∡TSV = m∡RSV	3. ∡-addition Postulate
4 (2,3) $2x+9 + 3x-2 = 67$	4. Substitution
5 $5x+7 = 67$	5. combining like terms
6 $5x = 60$	6 Subtraction prop. of equality
7 $x = \frac{60}{5} = 12$	7 Division prop. of equality
∴ $x = 12$	

<u>Exercise #12</u> If $m\angle A = 27^\circ$ and $\angle A$ and $\angle B$ are complementary, find the measure of $\angle B$.

Given: m∡A = 27°
<A and <B = complementary

Find: m<B

Proof

1. <A and < B = complementary (given)
2. m<A + m<B = 90° (defin of compl. ∡'s)
3. 27° + m<B = 90° (substitution)
4. m<B = 90° − 27° (subtraction prop. of eq.)

∴ m<B = 63°