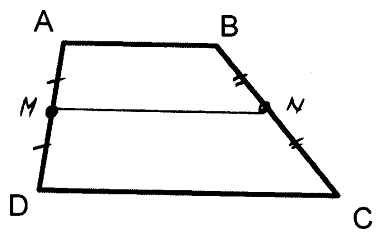
4.4 The Trapezoid

Definition A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. $\overline{AB} \parallel \overline{DC}$



Bases: $\overline{AB}$, $\overline{DC}$

Legs: $\overline{AD}$, $\overline{BC}$

Base angles: $\measuredangle D$ and $\measuredangle C$; $\measuredangle A$ and $\measuredangle B$

base $\measuredangle$'s exist in pairs

Median: $\overline{MN}$, M-midpoint $\overline{AD}$, N-midpoint $\overline{BC}$

Altitude: $\overline{AE}$, $\overline{CF}$

= line segment from one vertex of one base to the opposite base (or an extension of that base)

Questions: 1. Can you find any relationships between the angles of the trapezoid?

$m\angle A + m\angle D = 180°$ (because $\overline{AB} \parallel \overline{DC}$)

$m\angle B + m\angle C = 180°$

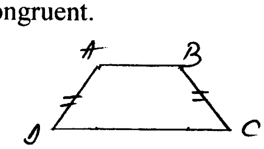
$m\angle A + m\angle B + m\angle C + m\angle D = 360°$

2. Can a trapezoid have all of its angles acute angles? Why or why not?

No. Then the sum of the angles would be less than 360°. (not possible)

Definition An **isosceles trapezoid** is a trapezoid with the nonparallel sides (legs) congruent.

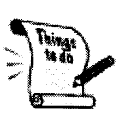
$\begin{cases} \overline{AB} \parallel \overline{CD} \\ \overline{AD} \cong \overline{BC} \end{cases}$

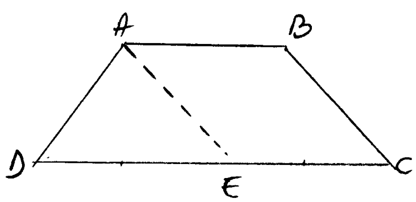


Properties of isosceles trapezoids

Theorem 1 (4.4 – T 4.20) $\boxed{\text{The base angles of an isosceles trapezoid are congruent.}}$

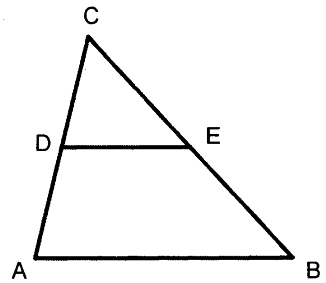
(base $\measuredangle$'s isosc. trap. $\cong$)





See textbook page 203

Problem #1 Use the figure to answer the questions.



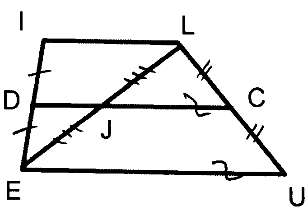
Given: D, E midpoints

a) What is DEBA?

b) If DE = 7 in, find AB.

c) If AB is 23 cm, find DE.

Problem #2 Use the figure to answer the questions.



Given: trap EULI ($\overline{EU}$, $\overline{IL}$ bases)

D, C midpoints, J midpoint $\overline{EL}$

$\overline{DC} \parallel \overline{EU}$

a) If IL = 43 cm, find DJ.

$\triangle EIL$: $DJ = \frac{1}{2} IL$

$= \frac{1}{2} 43 = 21.5$ cm

b) If EU = 17 in, find JC.

$\triangle LEU$: $JC = \frac{1}{2} EU$

$JC = \frac{1}{2} 17 = 8.5$ in

c) If JC = 12.5 cm, find EU.

$\triangle LEU$: $JC = \frac{1}{2} EU \Rightarrow$

$EU = 2JC = 2(12.5) = 25$ cm

e) If DJ = 6.3 cm, find IL.

$\triangle EIL$: $DJ = \frac{1}{2} IL$

$IL = 2DJ$

$= 2(6.3) = 13$ cm

f) If EU = 21 in and IL = 16 in, find DC.

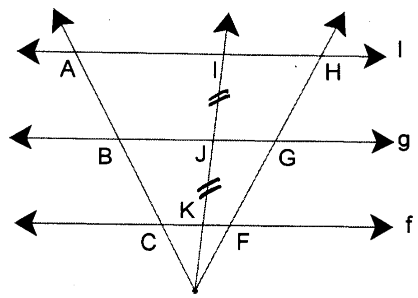
ILUE = trapezoid with $\overline{DC}$ - median

$DC = \frac{1}{2}(IL + EU)$

$DC = \frac{1}{2}(16+21) = \frac{1}{2}(37) = 18.5$ in

Problem #3 Use the figure to answer the questions.

if 3 ∥ lines cut ≅ segments on one transv, then ≅ segm. on any transverse



Given: $l \parallel g \parallel f$

$\overline{IJ} \cong \overline{JK}$

a) If AB = 14 cm, find AC.

$AB = BC = 14\text{ cm}$

$AC = AB + BC = 28\text{ cm}$

b) If FG = 3 in, find FH.

$FG = GH = 3\text{ in}$

$FH = 2\,GH = 6\text{ in}$

c) If AC = 36 cm, find BC.

$AC = AB + BC$ (b/c $BC = AB$)

$AC = 2\,BC$

$36 = 2\,BC \Rightarrow BC = 18\text{ cm}$

d) If GH = 22 in, find HF.

$GH = GF = 22\text{ in}$

$HF = 2\,GH$

$= 44\text{ in}$

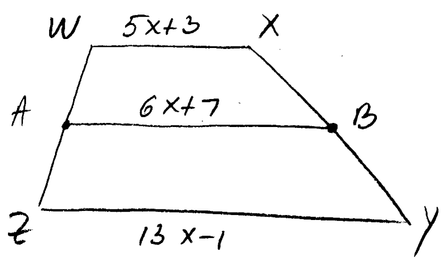
e) If BC = 4 in and GF = 6 in, find AC + HF.

$AC + HF = 2\,BC + 2\,GF$

$= 2(4) + 2(6)$

$= 8 + 12 = 20\text{ in}$

Problem #4 Let WXYZ a trapezoid with bases $WX = 5x + 3$ and $ZY = 13x - 1$. If the median $AB = 6x + 7$,
(4.4 - #32) find x.



Solution

AB - median $\Rightarrow$

$AB = \frac{1}{2}(WX + ZY)$

$2AB = WX + ZY$

$2(6x+7) = (5x+3) + (13x-1)$

$12x + 14 = 5x + 3 + 13x - 1$

$12x + 14 = 18x + 2$

$14 - 2 = 18x - 12x$

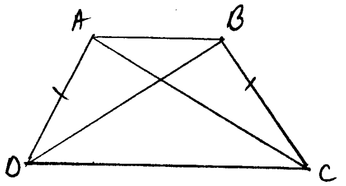
$12 = 6x$

$x = 6$

Corollary 1 (4.4 – t 4.21)

The diagonals of an isosceles trapezoid are congruent.

(diag. isosc. trap. $\cong$)



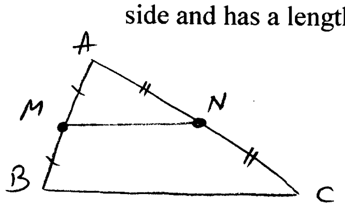
Given: ABCD isosc. trap. $\overline{AB} \parallel \overline{BC}$

Prove: $\overline{AC} \cong \overline{BD}$

Proof

1. ABCD - isos. trap.	1. given
2. $\overline{AD} \cong \overline{BC}$	2. def. isos. trap.
3. $\angle D \cong \angle C$	3. base $\angle$'s isos. trap. $\cong$
4. $\triangle ADC$, $\triangle BCD$: $\overline{DC} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$, $\angle D \cong \angle C$	4. reflexive prop. $\cong$; (2); (3)
5. $\triangle ADC \cong \triangle BCD$	5. SAS
6. $\overline{AC} \cong \overline{BD}$	6. CPCTC

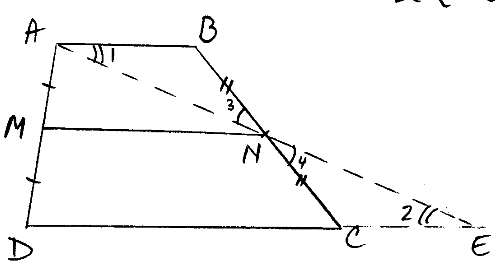
Recall : The segment that joins the midpoints of two sides of a triangle is parallel to the third side and has a length equal to half of the length of the 3rd side.



$\triangle ABC$
if M, N = midpoints, then $\overline{MN} \parallel \overline{BC}$, $MN = \frac{1}{2} BC$

Theorem 2 (4.4 – T 4.22)

The median of a trapezoid is parallel to each base. and $MN = \frac{1}{2}(AB + DC)$



Given: ABCD - trap, M - midpoint of $\overline{AD}$, N - midpoint of $\overline{BC}$

Prove: $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, $MN = \frac{1}{2}(BC + DC)$

Proof (informal)

Draw line $\overleftrightarrow{AN}$ and extend $\overrightarrow{DC}$

let $\overleftrightarrow{AN} \cap \overleftrightarrow{DC} = E$

$\triangle ANB \cong \triangle ENC$ (AAS):
- $\overline{BN} \cong \overline{CN}$ given
- $\angle 1 \cong \angle 2$ (alt. int. $\angle$'s formed by $\overline{AB} \parallel \overline{DC}$, transv. $\overline{AE}$)
- $\angle 3 \cong \angle 4$ (vertical $\angle$'s)

$\Rightarrow \overline{AN} \cong \overline{EN}$ and $\overline{AB} \cong \overline{CE}$

$\Rightarrow N$ = midpoint of $\overline{AE}$

in $\triangle ADE$, M - midpoint of $\overline{AD}$, N - midpoint of $\overline{AE}$ $\Rightarrow \overline{MN} \parallel \overline{DE}$

Therefore, $\overline{MN} \parallel \overline{DC} \parallel \overline{AB}$

Also, in $\triangle ADE$, $MN = \frac{1}{2} DE$

$MN = \frac{1}{2}(DC + CE)$

$MN = \frac{1}{2}(DC + AB)$

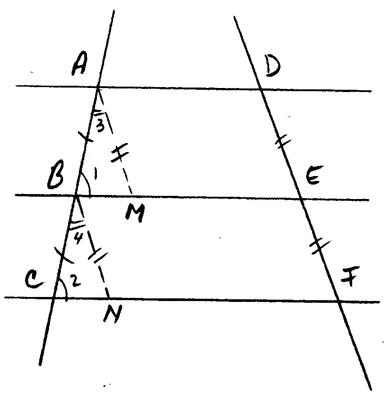
(see textbook #33/page 207)

Theorem (4.4 – T 4.23)

> If three (or more) parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on every transversal.

(if 3 || lines cut ≅ segm 1 trans, then ≅ segm every trans)

Write a formal proof .



Given: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$
$\overline{AB} \cong \overline{BC}$

Prove $\overline{DE} \cong \overline{EF}$

(see also textbook #34/page 208)

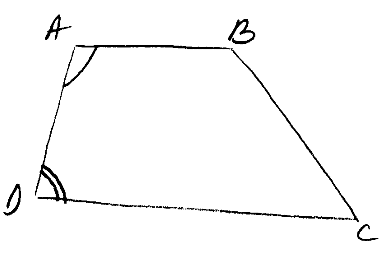
Proof

1. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$	1. given
2. $\angle 1 \cong \angle 2$	2. corresponding ∡'s ($\overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$ transv. $\overleftrightarrow{AC}$)
3. thru A draw $\overline{AM} \parallel \overline{DE}$ thru B draw $\overline{CN} \parallel \overline{DE}$	3. Parallel Postulate
4. $\overline{AM} \parallel \overline{BN}$ (3)	4. 2 lines // 3rd line are //
5. $\angle 3 \cong \angle 4$	5. corresponding ∡'s ($\overline{AM} \parallel \overline{BN}$, transv $\overleftrightarrow{AC}$)
6. $\triangle ABM$, $\triangle BCN$: $\overline{AB} \cong \overline{BC}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	6. given, (2), (5)
7. $\triangle ABM \cong \triangle BCN$	7. ASA
8. $\overline{AM} \cong \overline{BN}$	8. CPCTC
9. AMED - parallelogram	9. ▱ iff. opp. sides //
10. $\overline{AM} \cong \overline{DE}$	10. opp. sides ▱ ≅
11. BEFN - parallelogram	11. ▱ iff. opp. sides //
12. $\overline{BN} \cong \overline{EF}$	12. opp. sides ▱ ≅
13 $\overline{DE} \cong \overline{EF}$ (8, 10, 12)	13 transitivity

When is a quadrilateral a trapezoid?

Theorem 1

> If two of three consecutive angles of a quadrilateral are supplementary, the quadrilateral is a trapezoid.



Given: ABCD - quadrilateral
$\angle A$ and $\angle D$ = supplementary
$\angle D$ and $\angle C$ = not supplem.

Prove: ABCD - trapezoid

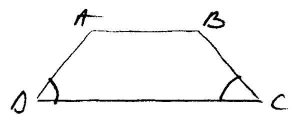
Condition: $\overline{AB} \parallel \overline{DC}$

Proof

1. ABCD - quadrilateral, $\angle A$ and $\angle D$ = supplem.	1. given
2. $\overline{AB} \parallel \overline{DC}$	2. // iff. int. ∡'s same side ≅ ($\overline{AB}$ and $\overline{DC}$ with transversal $\overline{AD}$)
3. ABCD - trapezoid	3. def. of trap

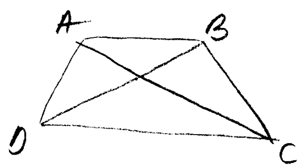
When is a trapezoid isosceles?

Theorem 1 If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.



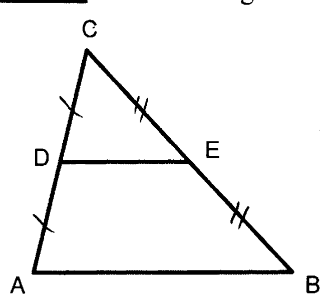
if ABCD - trap. with $\angle D \cong \angle C$ then ABCD - isosceles ($\overline{AD} \cong \overline{BC}$)

Theorem 2 If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.



if ABCD - trapezoid with $\overline{AC} \cong \overline{BD}$, then ABCD - isosceles ($\overline{AD} \cong \overline{BC}$)

Problem #1 Use the figure to answer the questions.



Given: D, E midpoints $\Rightarrow$ in $\triangle CAB$, $\overline{DE} \parallel \overline{AB}$ and $DE = \frac{1}{2} AB$

a) What is DEBA?

trapezoid b/c $\overline{DE} \parallel \overline{AB}$

b) If DE = 7 in, find AB.

$DE = \frac{1}{2} AB \Rightarrow AB = 2DE$

$AB = 2(7) = 14 \text{ in}$

c) If AB is 23 cm, find DE.

$DE = \frac{1}{2} AB$

$DE = \frac{1}{2} 23 = 11.5 \text{ cm}$