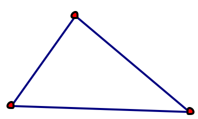
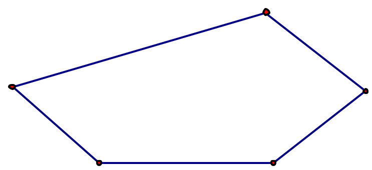
4.1 Parallelograms

Definition (3.3) A **polygon** is a closed figure whose sides are line segments that intersect only at endpoints. (*Polygon* is a word of Greek origin that means *many angles*; hence, it implies *many sides*).

Note: 1. We will be working only with **convex polygons,** polygons in which a line segment joining two points in the interior of the polygon has all its points in the interior of the polygon.
2. The angle measures of convex polygons are between 0° and 180°.

Examples of convex polygons





Definition (3.3) A **diagonal of a polygon** is a line segment that joins two nonconsecutive vertices.

Exercise #1 How many diagonals are in a

a) triangle

b) polygon with 4 sides (quadrilateral)

c) polygon with 5 sides (pentagon)

d) polygon with 6 sides (hexagon)

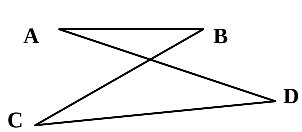
Definition (3.3) A **regular polygon** is a polygon with all its sides congruent and all its angles congruent.

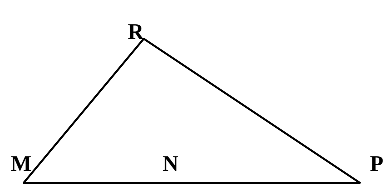
Definition (4.1) A **quadrilateral** is a polygon that has four sides.

Note: - We will work only with quadrilaterals whose sides are coplanar.
- Special quadrilaterals (squares, rectangles, rhombuses, parallelograms, and trapezoids) occur in various practical circumstances, such as architectural design, construction materials, fabric design, and urban planning.

Important! ABCD is a quadrilateral iff all points are coplanar, no three of which are collinear, and each segment intersects exactly two others, one at each endpoint.

Therefore, the following figures are **not** quadrilaterals:



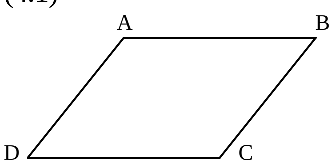


Property

The sum of the interior angles of a quadrilateral is 360°.

Definition
(4.1)

A **parallelogram** is a quadrilateral whose opposite sides are parallel.



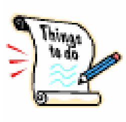
The defining property for a parallelogram is that it si a quadrilateral whose opposite sides are parallel. Many other properties follow from this. The most significant feature of the figure is that for either pair of opposite sides, the other two sides and the diagonals are transversals. Thus, the theory of parallel lines and transversals may be used to prove properties of paralle lograms. This theory and that for congruent triangles provide the needed tools for study of parallelograms.

Theorem 1
(4.1 – T 4.1)

A diagonal of a parallelogram separates it into two congruent triangles.

Properties of Parallelograms

Corollaries
(4.1 – C 4.2,3,4)



1. The opposite sides of a parallelogram are congruent (opp sides $\cong$).

2. The opposite angles of a parallelogram are congruent (opp $\angle' s$ $\cong$).

3. Any two consecutive angles of a parallelogram are supplementary (consec $\angle' s$ supp).

4. The diagonals of a parallelogram bisect each other (diags bisect each other).

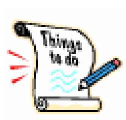
Summary: Properties of parallelograms

1. The opposite sides of a parallelogram are parallel.
2. Diagonal divides a parallelogram into two congruent triangles .
3. Any two consecutive angles of a parallelogram are supplementary.
4. The diagonals of a parallelogram bisect each other.
5. Opposite sides are congruent.
6. Opposite angles congruent.

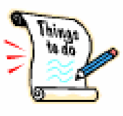
When is a quadrilateral a parallelogram?

Theorem3
(4.1 – T 4.6)

If two sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



Theorem 4
(4.1 – T 4.5)



If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.

Theorem 5
(4.1– T 4.8)

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 6
(4.1– T 4.6)

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram

Summary: Methods to prove a quadrilateral is a parallelogram

1. Show both pairs opposite sides are parallel.
2. Show both pairs of opposite sides are congruent.
3. Show both pairs of opposite angles are congruent.
4. Show one pair of opposite sides are congruent and parallel.
5. Show diagonal bisect each other.