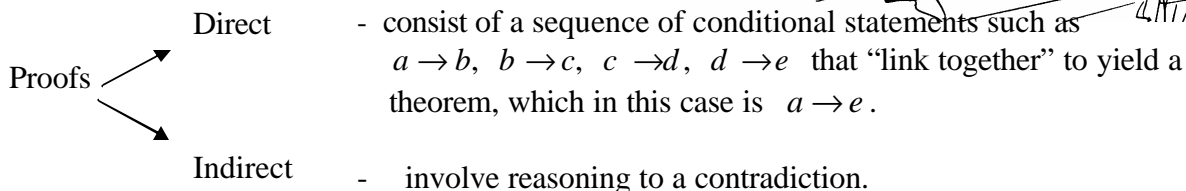


Section 3.1 Indirect Proof

A series of lessons in a subject that contradicted each other would make that subject very confusing. Yet, in reasoning deductively in geometry, it is sometimes helpful to try to arrive at contradictions deliberately!



Consider the following puzzle:

Emerson, Lake, and Palmer are different heights. Who is the tallest and who is the shortest if *only one* of the following statements is true?

1. Emerson is the tallest.
2. Lake is not the tallest.
3. Palmer is not the shortest.

One way of starting to figure this puzzle out would be to assume that the first statement is the one that is true, which would mean that the other two are false. This results in the following three statements:

1. Emerson is the tallest.
2. _____
3. _____

What is wrong? _____

Therefore, it is reasonable to conclude that, if there is nothing wrong with the puzzle itself, our assumption that the first statement is true must be wrong. Therefore, it is either the second or the third statement that is true.

On the assumption that the second statement is the true one, we get:

1. _____
2. _____
3. _____

What is the contradiction? _____

So, our assumption that the second statement is true is wrong.

That leaves just one more possibility if the puzzle has a solution at all; namely, that the third statement is the true one. We have:

1. _____
2. _____
3. _____

This time there is no contradiction. Evidently Lake is the tallest of the three and Emerson is the shortest.

The basic pattern in proving a theorem, say $P \rightarrow Q$, indirectly is to begin by assuming *not* Q ($\sim Q$). It is by reasoning from *this* statement that we hope to arrive at a contradiction. The statements Q and $\sim Q$ are called opposites of each other.

Exercise #1

Write the opposites (negation) of the following statements:

- a) Seven is a prime number. _____
- b) Mr. Spock does not like contradictions. _____
- c) A line contains at least two points. _____

To prove a theorem indirectly, we begin by assuming the opposite of its conclusion.

Exercise #2

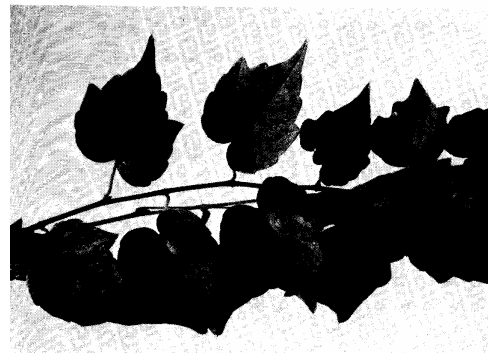
Write the opposite of the conclusion of each of the following theorems.

- a) If a number is odd, its square is odd. _____
- b) If two lines intersect, they intersect in no more than one point.

- c) In a plane, two lines perpendicular to a third line are parallel to each other.

Example of an indirect proof.

It is sufficient to know that poison ivy has leaves in groups of three to prove that the plant in this photograph is not poison ivy.



- a) With what assumption would we begin the proof?

- b) What conclusion follows from this assumption?

- c) What does this conclusion contradict?

Since our initial assumption led to a contradiction, it must be false. In other words, the statement “The plant in this photograph is poison ivy” is false.

- d) What statement, then, must be true? _____

Method of Indirect Proof

To prove the statement $P \rightarrow Q$ or to complete the proof problem of the form

Given: P

Prove: Q

where Q may be a negation, use the following steps:

1. Suppose that $\sim Q$ is true.
2. Reason from the supposition until you reach a contradiction.
3. Note that the supposition claiming that $\sim Q$ is true must be false and that Q must therefore be true.

Step 3 completes the proof.

Note that the contradiction that is discovered in an indirect proof often has the form $\sim P$. Thus the assumed statement $\sim Q$ has forced the conclusion $\sim P$, asserting that $\sim Q \rightarrow \sim P$ is true. Then the desired theorem $P \rightarrow Q$ (the contrapositive of $\sim Q \rightarrow \sim P$) is also true.

Exercise #3 | If a and b are positive numbers, then $\sqrt{a^2 + b^2} \neq a + b$.

Given:

Prove:

Proof:

Exercise #4 | The midpoint of a line segment is unique.

Given:

Prove:

Proof:

3.1 Parallel Lines

Definition **Parallel lines** are lines that lie in the same plane but do not intersect.

Postulate

If two lines intersect, they intersect at one point.

Postulate
(3.1 – P 3.1)

For a given line and a given point not on the line, there is one and only one line through the given point parallel to the given line.

Theorem
(3.1 – T 3.1)

If two coplanar lines are each perpendicular to a third line, then these lines are parallel to each other.

Hypothesis:

Conclusion:

Proof:

Exercise #5 Answer the following:

(3.1 - # 5 - 14)

a) Is $l \parallel m$?

b) Is $s \parallel n$?

c) Is $\overline{AB} \parallel \overline{ED}$?

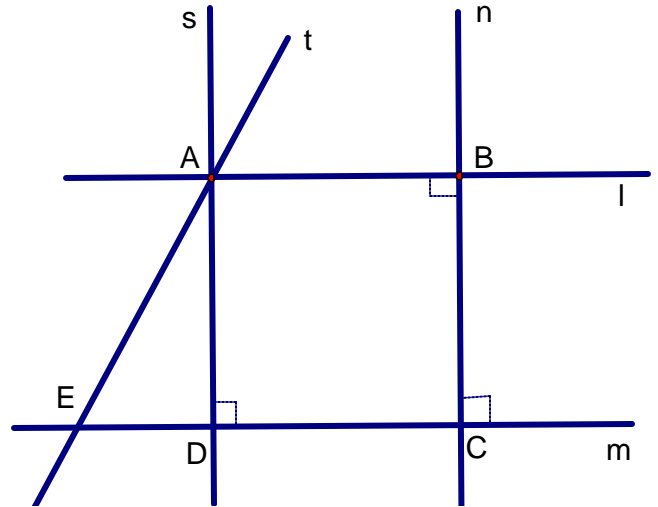
d) Is $\overline{AD} \parallel \overline{EC}$?

e) Can t be parallel to n ?

f) Does there exist a line through E parallel to n ?

g) How many lines through B are parallel to m ?

h) Is $\overline{AB} \parallel \overline{EC}$?



Exercise #6 If two lines are parallel to a third line then they are parallel to each other.

(3.1 - # 15)