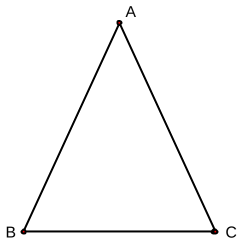
# 2.4 Isosceles Triangles
## Special Line Segments and Triangles

**Definition** A <u>triangle is isosceles</u> if and only if it has two congruent sides.

**Isosceles Triangle**




$\triangle ABC$ - isosceles

$\overline{AB} \cong \overline{AC}$

$\overline{BC}$ = base

$\angle A$ = vertex

$\angle B, \angle C$ = base angles

**Theorem** (2.4– T. 2.5) If two sides of a triangle are congruent, then the angles opposite the congruent sides are congruent.

**Theorem** (Converse of Theorem 2.5)
(2.4 – T. 2.7) If two angles of a triangle are congruent, then the sides opposite the congruent angles are congruent.

In conclusion, a triangle is isosceles if and only if ____________________________________________________

**Definition** A triangle is equilateral if and only if all three of its sides are congruent.

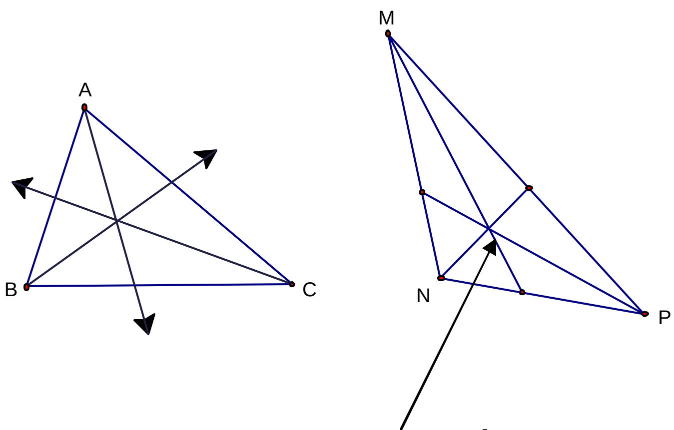
**Theorem** An equilateral triangle is also equiangular.
(2.4 – C. 2.6)

**Theorem** (Converse of Corollary 2.6)
(2.4 – C. 2.8) An equiangular triangle is also equilateral.

In conclusion, a triangle is equilateral if and only if ______________________________________________

**Special Line Segments**

**Definition** The bisectors of the angles of a triangle are called the **angle bisectors of a triangle**.



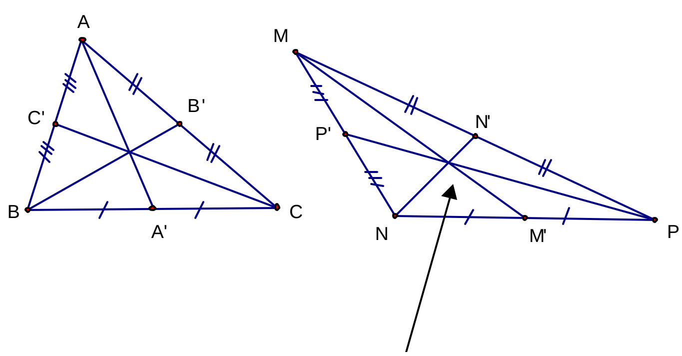

Note that the bisectors are shown as rays in the first figure and as line segments with endpoints on the triangle's sides in the second figure.

Note that the bisectors always intersect at one point in the interior of the triangle (concurrent lines).

**Incenter** = the intersection point of the bisectors of a triangle

**Theorem** (2.4 – T 2.12) The bisectors of the angles of a triangle are concurrent and meet at a point equidistant from the sides of the triangle.

**Definition** **A median of a triangle** is the segment joining a vertex and the midpoint of the opposite side.



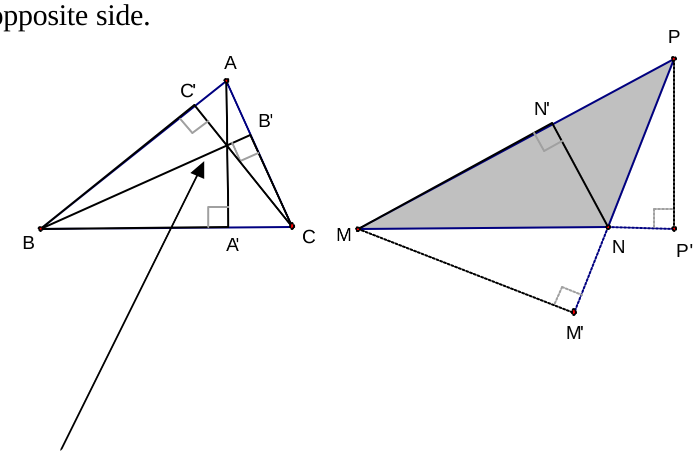

Note that the medians always intersect at one point in the interior of the triangle (concurrent lines).

Note that a median is not, in general, the angle bisector. Only in special cases do they coincide.

**Centroid** = the intersection point of the medians of a triangle

**Theorem** (2.4 – T 2.10) The medians of a triangle are concurrent and meet at a point that is two-thirds the distance from the vertex to the midpoint of the opposite side.

**Definition** An **altitude of a triangle** is a line segment from one vertex perpendicular to the line containing the opposite side.

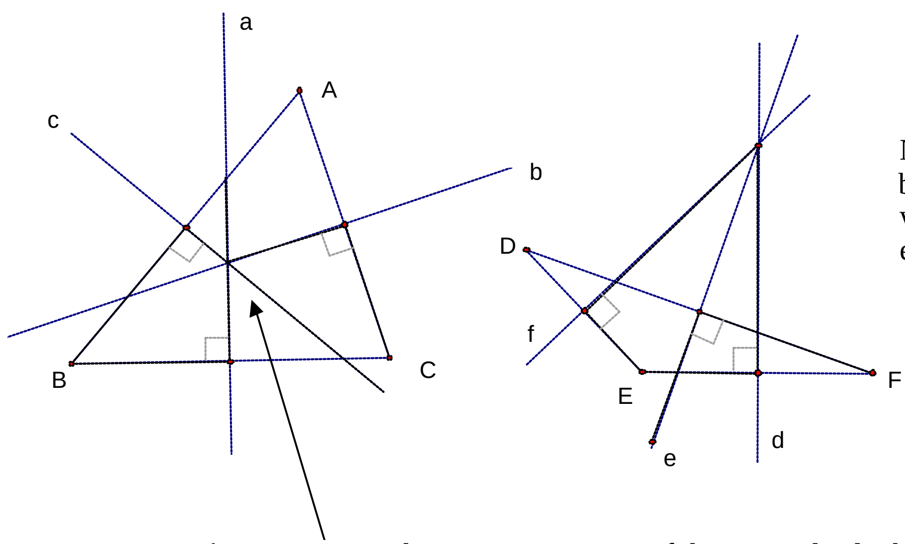



Note that an altitude does not always lie in the interior of a triangle.

Note that the altitudes of the first triangle intersect at one point. The altitudes of the second triangle would also intersect at one point if they were extended.

**Orthocenter** = the intersection point of the altitudes of a triangle

**Definition** **A perpendicular bisector of a side of a triangle** is the line that perpendicularly bisects the side of the triangle.




Note that the perpendicular bisectors always meet at a point which can be in the interior or exterior of the triangle.

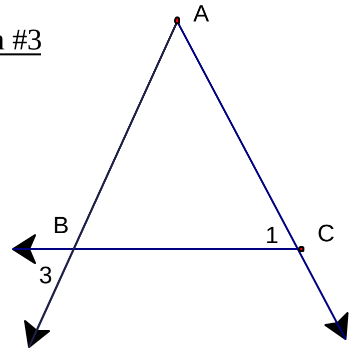
**Circumcenter =** the intersection point of the perpendicular bisectors

Auxiliary Lines Some proofs in geometry require the addition of lines, line segments, or rays to the given figure. These are called auxiliary lines (helping lines). Their relation to the given figure must be clearly stated and justified in the proof. You must account for the uniqueness of the line, segment or ray as it is introduced into the existing drawing.

Problem #1 In an isosceles triangle, one of the base angles is $68^\circ$. Find the other two angles of the triangle. Write a formal proof.

Problem #2 In an isosceles triangle ABC with vertex A, each base angle is 12 degrees larger than the vertex angle. Find the measure of each angle.

Problem #3




Given $\angle 3 \cong \angle 1$

Prove $\overline{AB} \cong \overline{AC}$

Problem #4 (2.4 - #12)

Let *ABE* an isosceles triangle with base *BE*. Let *C* and *D* two points on *BE* such that *B*-*C*-*D*-*E* and $\overline{BC} \cong \overline{DE}$. Show that $\angle ACD \cong \angle ADC$.

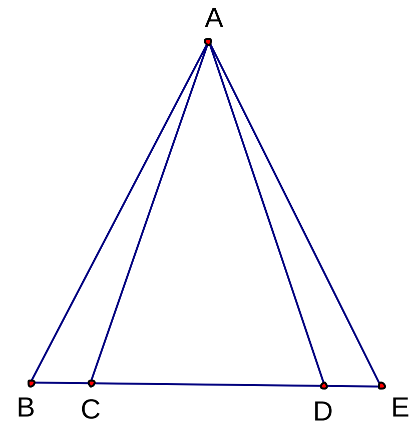
Problem #5

( 2.4 - #10)

Given: $\overline{AC} \cong \overline{AD}$

$\overline{BD} \cong \overline{CE}$

Prove: $\overline{AB} \cong \overline{AE}$




Problem #6 In an isosceles triangle ABC, M is the midpoint of the base $\overline{BC}$. Prove that $\triangle ABM \cong \triangle ACM$. What conlcuions could be drawn about the line segment $\overline{AM}$ ?