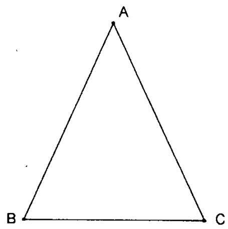
2.4 Isosceles Triangles
Special Line Segments and Triangles

Definition A <u>triangle is isosceles</u> if and only if it has two congruent sides.

Isosceles Triangle



$\triangle ABC$ - isosceles

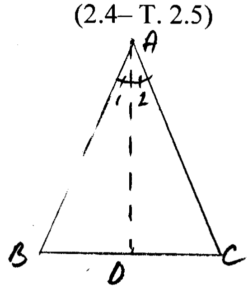
$\overline{AB} \cong \overline{AC}$

$\overline{BC}$ = base

$\angle A$ = vertex

$\angle B, \angle C$ = base angles

Theorem (2.4– T. 2.5) If two sides of a triangle are congruent, then the angles opposite the congruent sides are congruent.



Given: $\triangle ABC$, $\overline{AB} \cong \overline{AC}$

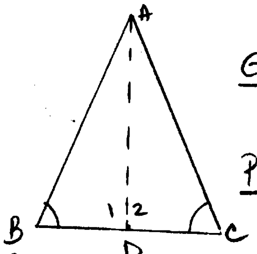
Prove: $\angle B \cong \angle C$

Proof

Statements	Reasons
1. draw $\overline{AD}$ - bisector of $\angle A$ with $D \in \overline{BC}$	1. The bisector of an angle is unique
2. $\angle 1 \cong \angle 2$	2. definition of $\angle$ bisector.
3. $\triangle ABD$, $\triangle ACD$: $\overline{AD} \cong \overline{AD}$, $\overline{AB} \cong \overline{AC}$, $\angle 1 \cong \angle 2$	3. reflexive prop. of $\cong$; given; (2) above
4. $\triangle ABD \cong \triangle ACD$	4. SAS
5. $\angle B \cong \angle C$	5. CPCTC

(Note: instead of drawing $\overline{AD}$ - bisector, we could draw $\overline{AD}$ - median. Then, $\triangle ABD \cong \triangle ACD$ by SSS)

Theorem (Converse of Theorem 2.5) (2.4 – T. 2.7) If two angles of a triangle are congruent, then the sides opposite the congruent angles are congruent.



Given: $\triangle ABC$, $\angle B \cong \angle C$

Prove: $\overline{AB} \cong \overline{AC}$

Proof

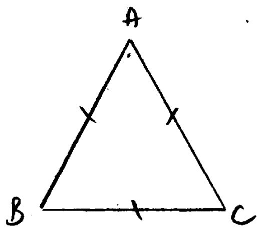
Statements	Reasons
1. Draw $\overline{AD} \perp \overline{BC}$, $D \in \overline{BC}$	1. The $\perp$ from a point to a line is unique.
2. $\angle D_1 \cong \angle D_2$	2. definition of $\perp$ lines. ($\perp$ iff $\cong$ adj. $\angle$'s)
3. $\triangle ABD$, $\triangle ACD$: $\overline{AD} \cong \overline{AD}$, $\angle B \cong \angle C$, $\angle D_1 \cong \angle D_2$	3. reflexive prop. of $\cong$; given; (2) above
4. $\triangle ABD \cong \triangle ACD$	4. AAS
5. $\overline{AB} \cong \overline{AC}$	5. CPCTC

(Note
- instead of constructing $\overline{AD}$ = altitude, we could also construct $\overline{AD}$ = bisector of $\angle A$ (then AAS)
- Also, we could show $\triangle ABC \cong \triangle ACB$)

In conclusion, a triangle is isosceles if and only if it has 2 congruent angles.

Definition A <u>triangle is equilateral</u> if and only if all three of its sides are congruent.

Theorem (2.4 – C. 2.6) An equilateral triangle is also equiangular.



Given: $\triangle ABC$ equilateral

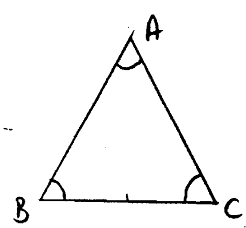
Prove: $\triangle ABC$ - equiangular

(Condition: $\angle A \cong \angle B \cong \angle C$)

Proof

Statements	Reasons
1. $\triangle ABC$ equilateral	1. given
2. $\overline{AB} \cong \overline{AC}$	2. Definition of equil. $\triangle$
3. $\angle C \cong \angle B$	3. $\triangle$, if 2 sides $\cong$, opp. $\angle$'s $\cong$.
4. $\overline{AB} \cong \overline{BC}$	4. Definition of equil. $\triangle$
5. $\angle C \cong \angle A$	5. Same as (3)
6. $\angle B \cong \angle A$ (3,5)	6. Transitivity $\cong$
7. $\triangle ABC$ equiangular	7. Definition of equiangular $\triangle$.

Theorem (Converse of Corollary 2.6) (2.4 – C. 2.8) An equiangular triangle is also equilateral.



Given: $\triangle ABC$ equiangular

Prove: $\triangle ABC$ equilateral

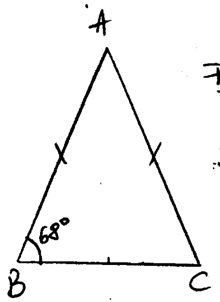
(Condition: $\overline{AB} \cong \overline{BC} \cong \overline{AC}$)

Proof

Statements	Reasons
1. $\triangle ABC$ equiangular	1. given
2. $\angle A \cong \angle B$	2. Definition of equiangular $\triangle$
3. $\overline{BC} \cong \overline{AC}$	3. $\triangle$, if 2 $\angle$'s $\cong$, opp. sides $\cong$.
4. $\angle B \cong \angle C$	4. Same as (2)
5. $\overline{AC} \cong \overline{AB}$	5. Same as (3)
6. $\overline{BC} \cong \overline{AB}$ (3,5)	6. Transitivity $\cong$
7. $\triangle ABC$ equilateral	7. Definition of equilateral $\triangle$.

In conclusion, a triangle is equilateral if and only if <u>it has three congruent angles.</u>

Problem #1 In an isosceles triangle, one of the base angles is 68°. Find the other two angles of the triangle.



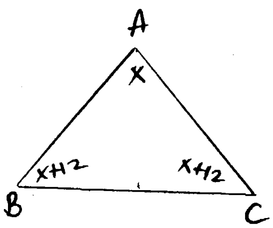
Given $\triangle ABC$ isosc.
$m\angle B = 68^\circ$

Find $m\angle A = ?$
$m\angle C = ?$

Proof

Statements	Reasons
1. $\triangle ABC$ isosceles	1. given
2. $\overline{AB} \cong \overline{AC}$	2. Definition of isosc. $\triangle$
3. $\angle C \cong \angle B$	3. $\triangle$, if 2 sides $\cong$, opp. $\angle$'s $\cong$
4. $m\angle C = m\angle B$	4. Definition of $\cong$ $\angle$'s.
5. $m\angle B = 68^\circ$	5. given
6. $m\angle C = 68^\circ$ (4,5)	6. transitivity
7. $m\angle A + m\angle B + m\angle C = 180^\circ$	7. $\triangle$, sum $\angle$'s $= 180^\circ$
8. $m\angle A + 68^\circ + 68^\circ = 180^\circ$	8. Substitution
9. $m\angle A = 44^\circ$	9. Subtraction prop. $=$

Problem #2 In an isosceles triangle ABC with vertex A, each base angle is 12 degrees larger than the vertex angle. Find the measure of each angle.

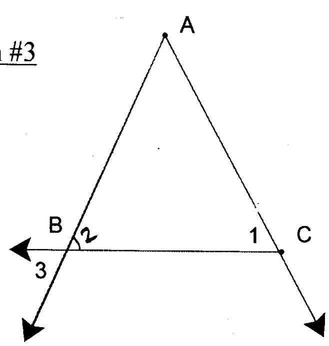


Let $m\angle A = x$
then $m\angle B = x+12$
$m\angle C = x+12$

In $\triangle ABC$, $m\angle A + m\angle B + m\angle C = 180^\circ$
$x + x + 12 + x + 12 = 180$
$3x + 24 = 180$
$3x = 156$
$x = 52$

Therefore, $m\angle A = 52^\circ$
$m\angle B = 52^\circ + 12^\circ = 64^\circ$
$m\angle C = 64^\circ$

Problem #3



Given $\angle 3 \cong \angle 1$

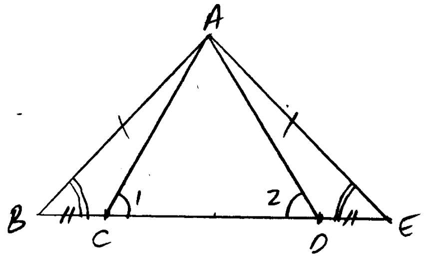
Prove $\overline{AB} \cong \overline{AC}$

Proof

Statements	Reasons
1. $\angle 3 \cong \angle 1$	1. given
2. $\angle 3 \cong \angle 2$	2. vertical angles
3. $\angle 1 \cong \angle 2$ (1,2)	3. transitivity
4. $\overline{AB} \cong \overline{AC}$	4. $\triangle ABC$, two $\angle$'s $\cong$, opp. sides $\cong$

Problem #4 (2.4 - #12)

Let ABE an isosceles triangle with base $\overline{BE}$. Let C and D two points on $\overline{BE}$ such that B-C-D-E and $\overline{BC} \cong \overline{DE}$. Show that $\angle ACD \cong \angle ADC$.



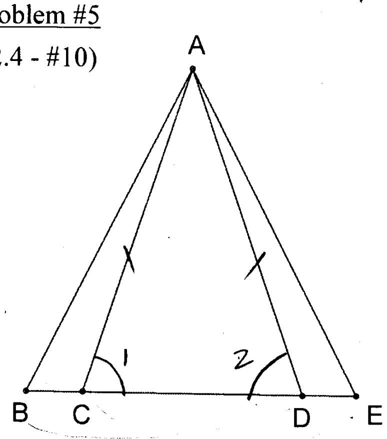
Given: $\triangle ABE$ isosceles
$\overline{BE}$ - base
$C, D \in \overline{BE}$, B-C-D-E
$\overline{BC} \cong \overline{DE}$

Prove: $\angle 1 \cong \angle 2$

Proof

1. $\triangle ABE$ isosceles, base $\overline{BE}$	1. given
2. $\overline{AB} \cong \overline{AE}$	2. def. of isosc. $\triangle$
3. $\angle E \cong \angle B$	3. in $\triangle ABE$, if 2 sides $\cong$, opposite $\angle$'s are $\cong$.
4. $\triangle ABC$, $\triangle AED$: $\overline{BC} \cong \overline{ED}$; $\angle B \cong \angle E$; $\overline{AB} \cong \overline{AE}$	4. given; (3) above; given
5. $\triangle ABC \cong \triangle AED$	5. SAS
6. $\overline{AC} \cong \overline{AD}$	6. CPCTC
7. $\angle 1 \cong \angle 2$	7. in $\triangle ACD$, if 2 $\angle$'s $\cong$, opp. sides $\cong$.

Problem #5 (2.4 - #10)



Given: $\overline{AC} \cong \overline{AD}$
$\overline{BD} \cong \overline{CE}$

Prove: $\overline{AB} \cong \overline{AE}$

Proof

Statements	Reasons
1. $\overline{AC} \cong \overline{AD}$	1. given
2. $\angle 1 \cong \angle 2$	2. $\triangle ACD$, if 2 sides $\cong$, opp $\angle$'s $\cong$.
3. $\triangle ABD$, $\triangle ACE$: $\overline{BD} \cong \overline{CE}$; $\angle 2 \cong \angle 1$; $\overline{AD} \cong \overline{AC}$	3. given; (2) above; given
4. $\triangle ABD \cong \triangle ACE$	4. SAS
5. $\overline{AB} \cong \overline{AE}$	5. CPCTC