1.4 Introduction to Deductive Proofs – Applications

Direct proofs

Example (1)

The chain events described in this ad might be expressed as follows:

A fish died
because
      it couldn't breathe
because
      its gills got clogged with silt
because
      mud ran into the river
because
      there was nothing to trap the rain
because
      there was a forest fire
because
      someone got careless with fire.

      So, please, be careful with fire…
because.

- If you are careless with fire, then there will be a
  _____.

- If there is a forest fire, then there will be
  _____.

- If there is _____,
  then the mud will run into the river.

- If mud runs into the river, then
  _____.

- If the gills of a fish get clogged with silt, then it can't breath.

- If a fish can't breathe, then a fish _____.

Therefore, _____

Example (2)

Theorem: *If a number is odd, then it can be written as* $2n+1$ *(where n is a whole number).*

Given (hypothesis): _____

Prove (conclusion): _____

Proof:

- If a number is odd, it is 1 more than _____.

- _____, it is 1 more than a multiple of 2.

- If a number is 1 more than a multiple of 2, it can be written as _____.

Exercise #1
(1.4 - example)

Prove the following theorem: *If I have enough money, then I will be unhappy.*

Assume that the following statements are true.

        Premise 1: If I have enough money, then I will take a trip.
        Premise 2: If I lose my job, I will be unhappy.
        Premise 3: If I take a trip, then I will lose my job.

Given: _____

Prove: _____

Proof:

Statements	Reasons

Exercise #2
(1.4 - #1)

Prove the following theorem:
If the president gets his budget passed, he will be voted out of the office.

Assume that the following statements are true.

        Premise 1: If taxes rise, then the people will be unhappy.
        Premise 2: If people are unhappy, the they will go to the polls.
        Premise 3: If the president gets his budget passed, then taxes will rise.
        Premise 4: If people go to the polls, the president will be voted out of the office.

Given: _____

Prove: _____

Proof:

Statements	Reasons

Exercise #3 (1.4 - #16, 21, 27, 28)	Given the following statements, do the following: 1. Identify the hypothesis and the conclusion. 2. Give the converse, inverse, and contrapositive.

a) If it rains, then I will stay indoors.

Hypothesis:

Conclusion:

Converse:

Inverse:

Contrapositive:

b) If a figure is a rectangle, then it is a parallelogram.

Hypothesis:

Conclusion:

Converse:

Inverse:

Contrapositive:

c) Vertical angles are congruent.

Hypothesis:

Conclusion:

Converse:

Inverse:

Contrapositive:

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d) Two angles are congruent if they are both right angles.

Hypothesis:

Conclusion:

Converse:

Inverse:

Contrapositive:

Exercise #4   Each of the following exercises consists of a "theorem" and a proof in which one or more of the statements has been omitted. By studying the relationships of the statements given, write the missing statements.

a)   Theorem:
        If you could count to one million without stopping, it would take you more than a week to do it.
    Proof:
        If you could count to one million without stopping, it would take you at least one million
    seconds.
        If something takes at least one million seconds, it takes more than sixteen thousand minutes.
        If something takes more than sixteen thousand minutes, it takes more than two hundred hours.

        (What statement belongs here?)

        If something takes more than eight days, it would take you more than a week to do it.

b)   Theorem:
        If there is a total eclipse of the sun, the temperature can be determined without a thermometer.
    Proof:

        (What statement belongs here?)

        If the sky becomes dark, crickets think it is night.

        (What statement belongs here?)

        If crickets start chirping, it is possible to estimate the temperature by counting the number of
    chirps per minute.
        If the temperature is estimated by counting cricket chirps, it can be determined without a
    thermometer.

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