

## 1.4 Introduction to Deductive Proofs – Applications

**Direct proofs**Example (1)

A fish died  
 because  
     it couldn't breathe  
 because  
     its gills got clogged with silt  
 because  
     mud ran into the river  
 because  
     there was nothing to trap the rain  
 because  
     there was a forest fire  
 because  
     someone got careless with fire.  
  
 So, please, be careful with fire...  
 because.

The chain events described in this ad might be expressed as follows :

- If you are careless with fire, then there will be a forest fire.
- If there is a forest fire, then there will be nothing to trap the rain.
- If there is nothing to trap the rain, then the mud will run into the river.
- If mud runs into the river, then the gills of the fish will get clogged with silt.
- If the gills of a fish get clogged with silt, then it can't breathe.
- If a fish can't breathe, then a fish will die.

Therefore, if you are careless with fire, then a fish will die.

Example (2)

Theorem: *If a number is odd, then it can be written as  $2n+1$  (where  $n$  is a whole number).*

Given (hypothesis): A number is odd

Prove (conclusion): it can be written as  $2n+1$

Proof:

- If a number is odd, it is 1 more than an even number.
- if a number is 1 more than an even number, it is 1 more than a multiple of 2.
- If a number is 1 more than a multiple of 2, it can be written as  $2n+1$ , where  $n$  is a whole #.

Exercise #1  
(1.4 - example)

Prove the following theorem: *If I have enough money, then I will be unhappy.*

Assume that the following statements are true.

Premise 1: If I have enough money, then I will take a trip.

Premise 2: If I lose my job, I will be unhappy.

Premise 3: If I take a trip, then I will lose my job.

Given: I have enough money.

Prove: I will be unhappy.

Proof:

Statements	Reasons
1. I have enough money	1. Given
2. I will take a trip	2. Premise (1)
3. I will lose my job.	3. Premise (3)
4. I will be unhappy.	4. Premise (2)
∴ if I have enough money, then I will be unhappy.	

Exercise #2  
(1.4 - #1)

Prove the following theorem:

*If the president gets his budget passed, he will be voted out of the office.*

Assume that the following statements are true.

Premise 1: If taxes rise, then the people will be unhappy.

Premise 2: If people are unhappy, then they will go to the polls.

Premise 3: If the president gets his budget passed, then taxes will rise.

Premise 4: If people go to the polls, the president will be voted out of the office.

Given: The president gets his budget passed

Prove: He will be voted out of the office.

Proof:

Statements	Reasons
1. The president gets his budget passed	1. Given
2. Taxes will rise	2. Premise (3)
3. People will be unhappy.	3. Premise (1)
4. People will go to the polls	4. Premise (2)
5. The president will be voted out of the office	5. Premise (4)
∴ if the president gets his budget passed, he will be voted out of the office.	

Exercise #3 | Given the following statements, do the following:  
 (1.4 - #16, 21, 27)

1. Identify the hypothesis and the conclusion.
2. Give the converse, inverse, and contrapositive.

a) If it rains, then I will stay indoors.

$$\boxed{p \rightarrow q}$$

Hypothesis: *It rains*

Conclusion: *I will stay indoors.*

Converse: *if I stay indoors, then it rains*

$$\boxed{q \rightarrow p}$$

Inverse: *if it doesn't rain, then I won't stay indoors*

$$\boxed{\sim p \rightarrow \sim q}$$

Contrapositive: *if I don't stay indoors, then it doesn't rain.*

$$\boxed{\sim q \rightarrow \sim p}$$

b) If a figure is a rectangle, then it is a parallelogram.

$$\boxed{p \rightarrow q}$$

Hypothesis: *A figure is a rectangle*

Conclusion: *The figure is a parallelogram*

Converse: *if a figure is a parallelogram, then it is a rectangle*

$$\boxed{q \rightarrow p}$$

Inverse: *if a figure is not a rectangle, then it's not a*

$$\boxed{\sim p \rightarrow \sim q}$$

Contrapositive: *if a figure is not a parallelogram, then*

*it is not a rectangle*

$$\boxed{\sim q \rightarrow \sim p}$$

c) Vertical angles are congruent.

Hypothesis: *Two angles are vertical.*

$$\boxed{p \rightarrow q}$$

Conclusion: *They are congruent*

Converse: *if two angles are congruent, then they are vertical*

$$\boxed{q \rightarrow p}$$

Inverse: *if two angles are not vertical, then they are not congruent*

$$\boxed{\sim p \rightarrow \sim q}$$

Contrapositive: *if two angles are not congruent, then they are not vertical*

$$\boxed{\sim q \rightarrow \sim p}$$