

## Circles Sections 6.1 - 6.4

The many practical uses of the circle range from the wheel to the near-circular orbits of some communication satellites. The mechanical uses of the circle have been known for thousands of years, and the ancient Greeks contributed significantly to our understanding of the circle's mathematical properties. The full moon, ripples in a pond when a stone is dropped in, and the shape of some bird's nests show some of the circles that appear in nature.

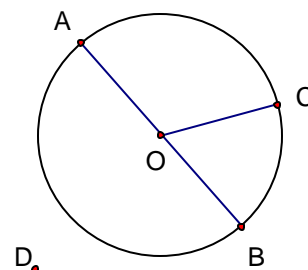
Our study of circles begins with some definitions, an explanation of the standard symbols used, and certain figures related to circles.

**Definition** (6.1) A **circle** is the set of all points in a plane that are at a given distance from a given point in the plane.

The given distance = \_\_\_\_\_

The given point = \_\_\_\_\_

Notation: \_\_\_\_\_



Note: A circle divides the plane into three distinct subsets:

- the interior \_\_\_\_\_
- the circle itself \_\_\_\_\_
- the exterior \_\_\_\_\_

Note: The radius of a circle is defined above as a number. It is standard practice, however, for “radius” to also mean a line segment, as in the following definition. You can usually determine which meaning of the word “radius” is intended by the context in which it is used.

**Definition** A **radius** of a circle is a segment that joins the center of the circle to a point on the circle. \_\_\_\_\_

**Definition** (6.1) A **diameter** of a circle is a segment whose endpoints are points of the circle and it contains the center of the circle. \_\_\_\_\_

**Theorem** In any given circle all radii are congruent and all diameters are congruent.  
(radii  $\odot \cong$  and diams  $\odot \cong$ )

**Postulate** (6.1 – P6.1) Two or more **circles are congruent** if and only if \_\_\_\_\_  
( $\odot s \cong$  iff radii  $\cong$ ).

**Definition** Two or more coplanar **circles are concentric** if \_\_\_\_\_

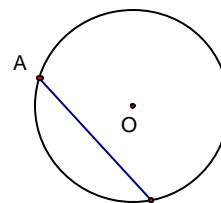
Question: How many circles can share the same center? \_\_\_\_\_

**Definition** (6.2) A line segment is a **chord** of a circle if its endpoints are points of the circle.

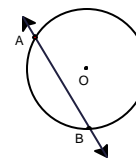
Questions:

- 1) Is a diameter a chord? \_\_\_\_\_
- 2) Is a radius of chord? \_\_\_\_\_
- 3) What is the longest possible chord? \_\_\_\_\_
- 4) How is the length of a chord related to its distance from the center? \_\_\_\_\_

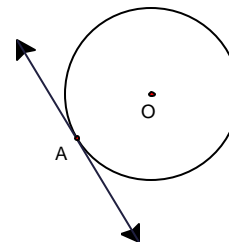
\_\_\_\_\_



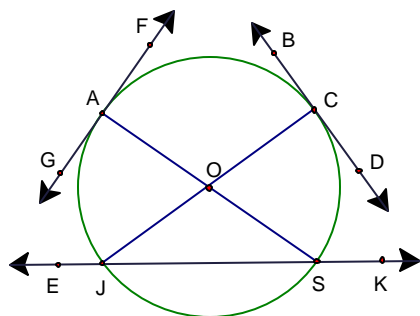
**Definition** (6.2) A line (or segment or ray) is a **secant** if it intersects a circle at exactly two points.



**Definition** (6.3) A line is a **tangent** to a circle if it intersects the circle at exactly one point.



**Problem #1** In the given figure, name:



- a) four radii
- b) two diameters
- c) three chords
- d) two tangents
- e) one secant

Several types of angles associated with circles are seen in the above figure. The next definition describes the most fundamental of these angles.

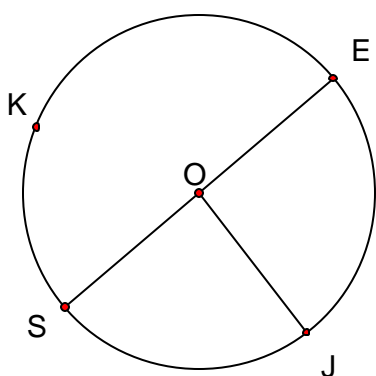
**Definition** (6.1) An angle is a **central angle** of a circle if \_\_\_\_\_

A central angle may be - acute \_\_\_\_\_

- right

- obtuse (measure less than  $180^\circ$ ) \_\_\_\_\_

These angles “cut off” portions of the circle called arcs.



**Definition** (6.1) A **minor arc** is the set of points of a circle that are on a central angle or in its \_\_\_\_\_.

Example: \_\_\_\_\_

**Definition** The **intercepted arc of an angle** is the minor arc associated with the central angle.

Example: What is the intercepted arc of  $\angle SOJ$  ?

**Definition** (6.1) A **major arc** is the set of points of a circle that are on a central angle or in its \_\_\_\_\_.

Example: \_\_\_\_\_

**Definition** (6.1) A **semicircle** is the set of points of a circle that are on, or are on one side of, a line containing a \_\_\_\_\_

Example: \_\_\_\_\_

### Arc Addition Postulate

(6.1) Let A, B, and C be three points on the same circle with B between A and C. Then

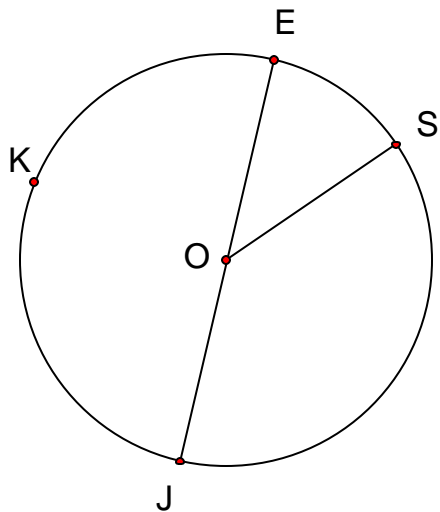
$$m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$$

**Definition**

The degree measure of

(6.1)

- a minor arc is the measure of its central angle (also known as **The Central Angle Postulate**),
- a semicircle is  $180^\circ$ ,
- a circle is  $360^\circ$ ,
- a major arc is  $360^\circ$  minus the measure of its associated minor arc.

**Problem #2**Given:  $\odot O$ 

$$m\angle EOS = 41^\circ$$

Find:  $m\widehat{ES}$ 

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$$m\widehat{ESJ}$$

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$$m\angle SOJ$$

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$$m\widehat{SJ}$$

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$$m\widehat{EKS}$$

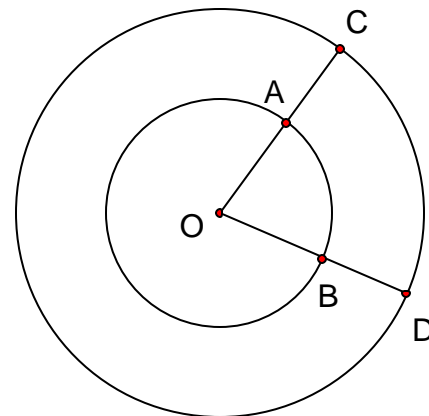
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Note: The degree measure of an arc is not a measure of the arc's length.

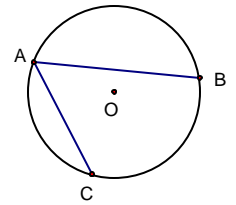
For the concentric circles in the figure,

$m\widehat{AB} = m\widehat{CD}$  because the arcs have the same central angle,

but certainly  $\widehat{AB}$  is not as long as  $\widehat{CD}$ .



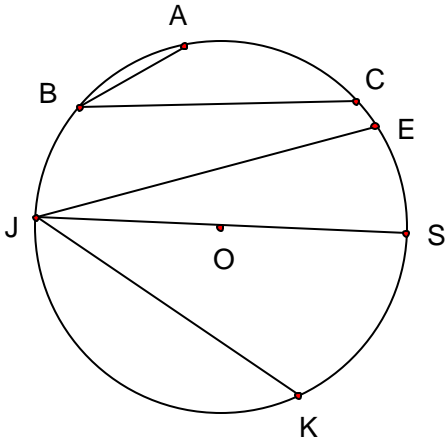
## Inscribed Angles (6.1)



**Definition**  
(6.1)

An angle is an **inscribed angle** of a circle if its vertex is a point on the circle and its sides are chords of the circle.

There are three different types of inscribed angles when considered in relation to the center of the circle.



- 1) One side of the angle may contain a diameter, as do \_\_\_\_\_
- 2) The circle's center may be in the angle's interior as is the case for \_\_\_\_\_
- 3) The center may be in the angle's exterior as it is for \_\_\_\_\_

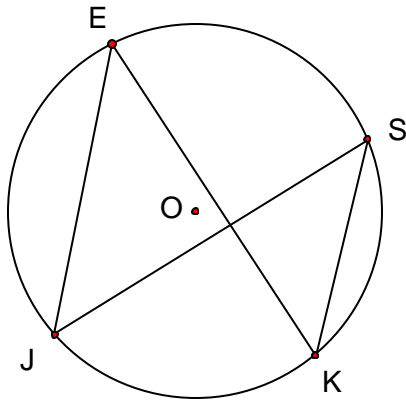
**Theorem 1**  
(6.1 – T 6.2)

The measure of an inscribed angle is equal to one-half the degree measure of its intercepted arc.  
(inscr  $\angle = \frac{1}{2} \widehat{\text{arc}}$ )

Example:

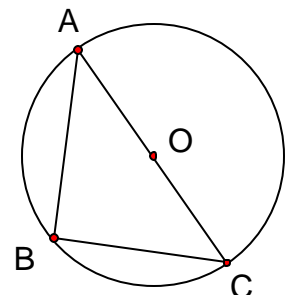
**Theorem 2**  
(6.1 – C 6.3)

If two inscribed angles in a circle intercept the same arc or congruent arcs, then the angles are congruent (inscr  $\angle$ s intercept same  $\widehat{\text{arc}}$  or  $\cong \widehat{\text{arc}}$ s are  $\cong$ ).



**Theorem 3**  
(6.1 – C 6.4)

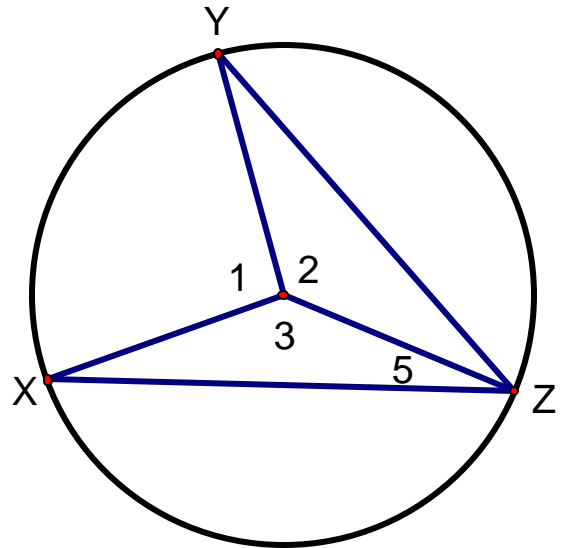
If an inscribed angle intercepts a semicircle, then it is a \_\_\_\_\_ angle  
(inscr  $\angle$  interc semi  $\odot$  is rt  $\angle$ ).



Problem #3 Use the figure to answer the questions.

(6.1 - # 21 - 24)  $m\widehat{XY} : m\widehat{YZ} : m\widehat{ZX} = 5 : 6 : 7$

a) Find  $m\widehat{XY}$ ,  $m\widehat{YZ}$ , and  $m\widehat{ZX}$ .



b) Find the measure of angles 1 to 5.

Problem #4 Use the figure to answer each question.

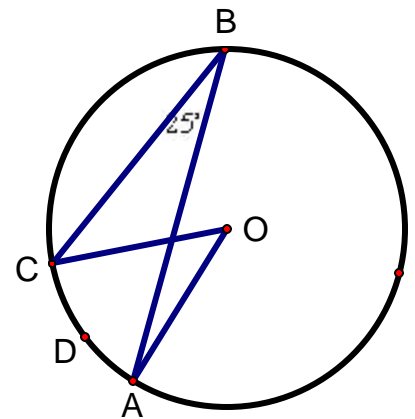
(6.1 - # 26)

a) What is  $m\angle AOC$

b) What is  $m\widehat{AC}$

c) What is  $m\widehat{ABC}$ ?

d) What is the measure of the arc intercepted by  $\angle ABC$ ?

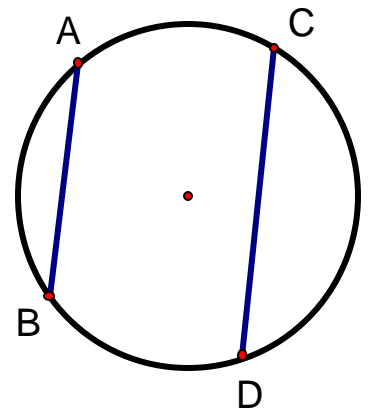


Problem # 5

Given:  $\overline{AB} \parallel \overline{CD}$

(6.1 - # 30)

Prove:  $m\widehat{AC} = m\widehat{BD}$



## Central Angles, Arcs, and Chords

### 6.2

There are some important properties about central angles, arcs, and chords that are associated with a given circle or with two circles that are the same size. But what is meant by “the same size”? \_\_\_\_\_

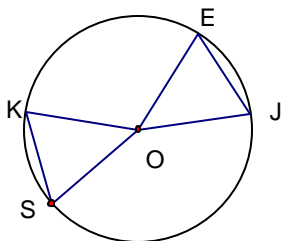
**Definition** Two **arcs** of a circle or of congruent circles are congruent iff their degree measures are equal.

**Note:** Since congruent arcs are defined in terms of numbers (degree measures), the addition, subtraction, multiplication, and division properties of congruence may be easily extended to include congruence between arcs.

**Theorem 4** If two chords in a circle or in congruent circles are congruent, then their arcs are congruent  
(6.2 – T 6.6) (if chords  $\cong$ ,  $\widehat{s} \cong$ ).



Write a formal proof.



**Theorem 5** (Converse of Theorem 4)

(6.2 – T. 6.7)

(if  $\widehat{s} \cong$ , chords  $\cong$ ).

Remember that the measure of a central angle is equal to the measure of its \_\_\_\_\_

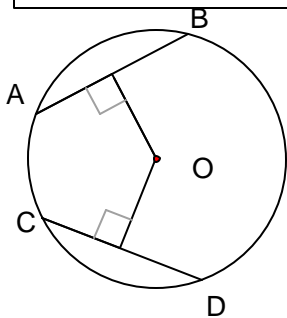
Therefore, we have the following property (theorem):

**Theorem 6** Two minor arcs of a circle or of congruent circles are congruent if and only if their central angles are congruent ( $\widehat{s} \cong \text{iff central } \angle s \cong$ ).

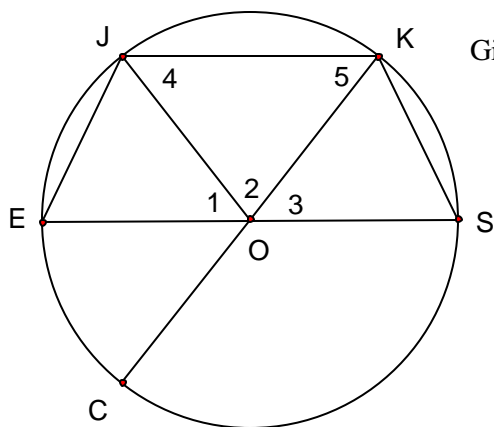
The above three theorems are summarized in the following diagram:

$\cong \text{ central angles } \leftrightarrow \cong \text{ arcs } \leftrightarrow \cong \text{ chords.}$

**Theorem 7** (6.2 – T. 6.10 and T. 6.11) Chords are at the same distance from the center of a circle if and only if they are congruent.



Problem #6



Given:  $\odot O$   
 $\overline{ES} \parallel \overline{JK}$   
 $m\widehat{JK} = 78^\circ$   
 $JE = 1.5 \text{ cm}$

Find:  
 a)  $\angle s 1-5$   
 b)  $m\widehat{JE}$ ,  $m\widehat{KS}$ ,  $KS$ ,  $m\widehat{JC}$