## Chapter 10 <br> Trigonometry of Right Triangles <br> Trigonometric Ratios - Applications

1. A giant redwood tree casts a shadow 532 ft long. Find the height of the tree if the angle of elevation of the sun is $25.7^{\circ}$
2. From a point on the ground 500 feet from the base of a building, it is observed that the angle of elevation to the top of the building is $24^{\circ}$ and the angle of elevation to the top of a flagpole atop the building is $27^{\circ}$. Find the height of the building and the length of the flagpole.
(A: 32 ft )
3. Find $\sin \alpha, \cos \beta, \tan \alpha, \cot \beta$ if


$$
\mathrm{A}:\left(\frac{2 \sqrt{10}}{7}, \frac{2 \sqrt{10}}{7}, \frac{2 \sqrt{10}}{3}, \frac{2 \sqrt{10}}{3}\right)
$$

4. Find the side labeled $x$.


$$
\left(A: \frac{13 \sqrt{3}}{2}, 4 \sqrt{3}\right)
$$

5. Find $x$ correct to five decimal places.

6. Sketch a right triangle that has one acute angle $\theta$, and find the other five trigonometric ratios of $\theta$.
a) $\sin \theta=\frac{3}{5}$
b) $\tan \theta=\sqrt{3}$
7. Express $x$ and $y$ in terms of trigonometric ratios of $\theta$.

$(x=28 \cos \theta, y=28 \sin \theta)$
8. Evaluate the expressions:
a) $\sin 30^{\circ}+\cos 30^{\circ}$
b) $\sin 30^{\circ} \csc 30^{\circ}$
c) $\left(\sin 60^{\circ}\right)^{2}+\left(\cos 60^{\circ}\right)^{2}$
9. Solve the right triangle.

10. Using a protractor, sketch a right triangle that has the acute angle $40^{\circ}$. Measure the sides carefully and use your results to estimate the six trigonometric ratios of $40^{\circ}$.
11. From the top of a $200-\mathrm{ft}$ lighthouse, the angle of depression to a ship in the ocean is $23^{\circ}$. How far is the ship form the base of the lighthouse?
(A: approx. 471 ft )
12. A 20 -ft ladder leans against a building so that the angle between the ground and the ladder is $72^{\circ}$. How high does the ladder reach on the building?
13. A man is lying on the beach, flying a kite. He holds the end of the kite string at ground level, and estimates the angle of elevation of the kite to be $50^{\circ}$. If the string is 450 ft long, how high is the kite above the ground?
14. A water tower is located 325 ft from a building. From a window in the building, it is observed that the angle of elevation to the top of the tower is $39^{\circ}$ and the angle of depression to the bottom of the tower is $25^{\circ}$. How tall is the tower? How high is the window?
15. An airplane flying at the rate of 350 feet per second begins to climb at an angle of $10^{\circ}$. What is the increase in altitude over the next 15 seconds if the speed remains the same?
(A: 912 ft )
16. At an altitude of $12,000 \mathrm{ft}$, a pilot sees two towns through angles of depression of $37^{\circ}$ and $48^{\circ}$. To the nearest ten feet, how far apart are the towns?
(A: 5121 ft )
17. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be $32^{\circ}$. One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is $35^{\circ}$. Estimate the height of the mountain.
(A: 4200 ft )
18. Find x correct to one decimal place.


## Simplifying Trigonometric Expressions <br> Proving Trigonometric Identities

Simplify the following expressions:

1. $(1+\sin x)(\sec x-\tan x)$
2. $\frac{\sin a}{\cos a}+\frac{\cos a}{1+\sin a}$
3. $\cos ^{3} y+\sin ^{2} y \cos y$
4. $\frac{\cos \theta \sec \theta}{\cot \theta}$
5. $\frac{\sec ^{2} x-1}{\sec ^{2} x}$
6. $\frac{\sin t}{\csc t}+\frac{\cos t}{\sec t}$
7. $\frac{1+\sin \alpha}{\cos \alpha}+\frac{\cos \alpha}{1+\sin \alpha}$
8. $\frac{2+\tan ^{2} x}{\sec ^{2} x}-1$
9. $\frac{1+\cot A}{\csc A}$

Answers:

1) $\cos x ; 2) \sec a ; 3) \cos y ; 4) \tan (; 5) \sin ^{2} x$; 6) 1 ; 7) $\frac{2}{\cos \alpha}=2 \sec \alpha$; 8) $\cos ^{2} x$; 9) $\sin \mathrm{A}+\cos \mathrm{A}$

Prove the following identities:

1. $\cos a(\sec a-\cos a)=\sin ^{2} a$
2. $2 \tan x \sec x=\frac{1}{1-\sin x}-\frac{1}{1+\sin x}$
3. $\frac{1}{1-\sin t}=\sec ^{2} t+\tan t \sec t$
4. $\frac{1+\cos \theta}{\cos \theta}=\frac{\tan ^{2} \theta}{\sec \theta-1}$
5. $\sin x \cot x=\cos x$
6. $\frac{\tan a}{\sec a}=\sin a$
7. $\frac{\cos t}{\sec t}+\frac{\sin t}{\csc t}=1$
8. $(\sin \theta+\cos \theta)^{2}=1+2 \sin \theta \cos \theta$
9. $\frac{1-\sin x}{1+\sin x}=(\sec x-\tan x)^{2}$
10. $\sin a+\cos a \cot a=\csc a$
