Chapter 10 Trigonometry of Right Triangles

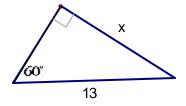
Trigonometric Ratios – Applications

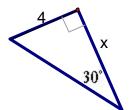
- 1. A giant redwood tree casts a shadow 532 ft long. Find the height of the tree if the angle of elevation of the sun is 25.7° (A: 256 ft)
- 2. From a point on the ground 500 feet from the base of a building, it is observed that the angle of elevation to the top of the building is 24° and the angle of elevation to the top of a flagpole atop the building is 27°. Find the height of the building and the length of the flagpole. (A: 32 ft)
- 3. Find $\sin a$, $\cos b$, $\tan a$, $\cot b$ if



A:
$$\left(\frac{2\sqrt{10}}{7}, \frac{2\sqrt{10}}{7}, \frac{2\sqrt{10}}{3}, \frac{2\sqrt{10}}{3}\right)$$

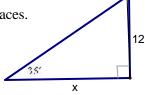
4. Find the side labeled x.

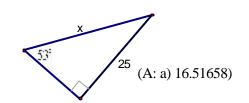




$$\left(A:\frac{13\sqrt{3}}{2},4\sqrt{3}\right)$$

5. Find *x* correct to five decimal places.



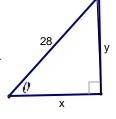


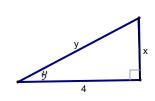
6. Sketch a right triangle that has one acute angle q, and find the other five trigonometric ratios of q.

a)
$$\sin q = \frac{3}{5}$$

b)
$$\tan q = \sqrt{3}$$

7. Express x and y in terms of trigonometric ratios of \boldsymbol{q} .





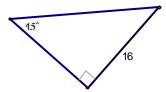
$$(x = 28\cos q, y = 28\sin q)$$

8. Evaluate the expressions:

a)
$$\sin 30^{\circ} + \cos 30^{\circ}$$

c)
$$(\sin 60^{\circ})^2 + (\cos 60^{\circ})^2$$

9. Solve the right triangle.

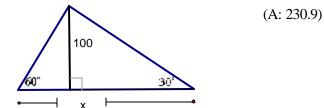


(A: leg=16, hyp=
$$16\sqrt{2}$$
)

- 10. Using a protractor, sketch a right triangle that has the acute angle 40° . Measure the sides carefully and use your results to estimate the six trigonometric ratios of 40° .
- **11**. From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocean is 23°. How far is the ship form the base of the lighthouse?

 (A: approx. 471 ft)
- **12.** A 20-ft ladder leans against a building so that the angle between the ground and the ladder is 72°. How high does the ladder reach on the building?
- 13. A man is lying on the beach, flying a kite. He holds the end of the kite string at ground level, and estimates the angle of elevation of the kite to be 50° . If the string is 450 ft long, how high is the kite above the ground?
- **14.** A water tower is located 325 ft from a building. From a window in the building, it is observed that the angle of elevation to the top of the tower is 39° and the angle of depression to the bottom of the tower is 25°. How tall is the tower? How high is the window?

 (A: 414 ft)
- **15.** An airplane flying at the rate of 350 feet per second begins to climb at an angle of 10° . What is the increase in altitude over the next 15 seconds if the speed remains the same? (A: 912 ft)
- **16.** At an altitude of 12,000 ft, a pilot sees two towns through angles of depression of 37° and 48° . To the nearest ten feet, how far apart are the towns? (A: 5121 ft)
- 17. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be 32° . One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is 35° . Estimate the height of the mountain. (A: 4200 ft)
- **18.** Find x correct to one decimal place.



Simplifying Trigonometric Expressions Proving Trigonometric Identities

Simplify the following expressions:

1.
$$(1+\sin x)(\sec x - \tan x)$$

$$4. \ \frac{\cos q \sec q}{\cot q}$$

$$7. \frac{1+\sin a}{\cos a} + \frac{\cos a}{1+\sin a}$$

$$2. \frac{\sin a}{\cos a} + \frac{\cos a}{1 + \sin a}$$

$$5. \frac{\sec^2 x - 1}{\sec^2 x}$$

8.
$$\frac{2+\tan^2 x}{\sec^2 x} - 1$$

$$3. \cos^3 y + \sin^2 y \cos y$$

$$6. \frac{\sin t}{\csc t} + \frac{\cos t}{\sec t}$$

9.
$$\frac{1+\cot A}{\csc A}$$

Answers:

1)
$$\cos x$$
; 2) $\sec a$; 3) $\cos y$; 4) $\tan q$; 5) $\sin^2 x$; 6) 1; 7) $\frac{2}{\cos a} = 2\sec a$; 8) $\cos^2 x$; 9) $\sin A + \cos A$

Prove the following identities:

1.
$$\cos a(\sec a - \cos a) = \sin^2 a$$

2.
$$2\tan x \sec x = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$$

3.
$$\frac{1}{1-\sin t} = \sec^2 t + \tan t \sec t$$

$$4. \frac{1+\cos \mathbf{q}}{\cos \mathbf{q}} = \frac{\tan^2 \mathbf{q}}{\sec \mathbf{q} - 1}$$

5.
$$\sin x \cot x = \cos x$$

$$6. \frac{\tan a}{\sec a} = \sin a$$

$$7. \frac{\cos t}{\sec t} + \frac{\sin t}{\csc t} = 1$$

8.
$$\left(\sin \mathbf{q} + \cos \mathbf{q}\right)^2 = 1 + 2\sin \mathbf{q} \cos \mathbf{q}$$

9.
$$\frac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$$

10.
$$\sin a + \cos a \cot a = \csc a$$