

Section 1.3 – Examples

EXAMPLE 1 Evaluating Expressions

Find the value of each algebraic expression when $m = 5$.

(a) $8m$

$$= 8 \cdot m$$

$$= 8 \cdot 5 \quad \text{Let } m = 5.$$

$$= 40 \quad \text{Multiply.}$$

(b) $3m^2$

$$= 3 \cdot m^2$$

$$= 3 \cdot 5^2 \quad \text{Let } m = 5.$$

$$= 3 \cdot 25 \quad \text{Square 5.}$$

$$= 75 \quad \text{Multiply.}$$

✓ Now Try Exercises 15 and 17.

▶ **CAUTION** In Example 1(b), $3m^2$ means $3 \cdot m^2$, not $3m \cdot 3m$. Unless parentheses are used, the exponent refers only to the variable or number just before it. To write $3m \cdot 3m$ with exponents, use parentheses: $(3m)^2$.

EXAMPLE 2 Evaluating Expressions

Find the value of each expression when $x = 5$ and $y = 3$.

(a) $2x + 7y$

$$= 2 \cdot 5 + 7 \cdot 3 \quad \text{Let } x = 5 \text{ and } y = 3.$$

$$= 10 + 21 \quad \text{Multiply.}$$

$$= 31 \quad \text{Add.}$$

(b) $\frac{9x - 8y}{2x - y}$

$$= \frac{9 \cdot 5 - 8 \cdot 3}{2 \cdot 5 - 3} \quad \text{Let } x = 5 \text{ and } y = 3.$$

$$= \frac{45 - 24}{10 - 3} \quad \text{Multiply.}$$

$$= \frac{21}{7}, \text{ or } 3 \quad \text{Subtract, then divide.}$$

(c) $x^2 - 2y^2$

$$= 5^2 - 2 \cdot 3^2 \quad \text{Let } x = 5 \text{ and } y = 3.$$

$$= 25 - 2 \cdot 9 \quad \text{Apply the exponents.}$$

$$= 25 - 18 \quad \text{Multiply.}$$

$$= 7 \quad \text{Subtract.}$$

EXAMPLE 3 Using Variables to Write Word Phrases as Algebraic Expressions

Write each word phrase as an algebraic expression, using x as the variable.

- (a) The
- sum**
- of a number and 9

“Sum” is the answer to an addition problem. The given phrase translates as

$$x + 9 \quad \text{or} \quad 9 + x.$$

- (b) 7
- minus**
- a number

“Minus” indicates subtraction, so the translation is

$$7 - x.$$

Note that $x - 7$ would not be correct, because we cannot subtract in either order and get the same result.

- (c) A number
- subtracted from 12**

Since a number is subtracted *from* 12, write this as

$$12 - x.$$

Compare this result with “12 subtracted from a number,” which is $x - 12$.

- (d) The
- product**
- of 11 and a number

$$11 \cdot x, \quad \text{or} \quad 11x$$

- (e) 5
- divided by**
- a number

$$5 \div x, \quad \text{or} \quad \frac{5}{x}$$

$\frac{5}{x}$ is not correct here.

- (f) The
- product**
- of 2 and the
- difference**
- between a number and 8

We are multiplying 2 times another number. This number is the difference between a number and 8, written $x - 8$. Using parentheses around this difference, we find that the final expression is

$$2(x - 8).$$

EXAMPLE 4 Deciding whether a Number Is a Solution of an Equation

Decide whether the given number is a solution of the equation.

- (a)
- $5p + 1 = 36$
- ; 7

$$\begin{array}{rcl} 5p + 1 = 36 & & \\ 5 \cdot 7 + 1 = 36 & ? & \text{Let } p = 7. \\ 35 + 1 = 36 & ? & \\ 36 = 36 & & \text{True} \end{array}$$

*Be careful!
Multiply first.*

The number 7 is a solution of the equation.

- (b)
- $9m - 6 = 32$
- ; 4

$$\begin{array}{rcl} 9m - 6 = 32 & & \\ 9 \cdot 4 - 6 = 32 & ? & \text{Let } m = 4. \\ 36 - 6 = 32 & ? & \\ 30 = 32 & & \text{False} \end{array}$$

The number 4 is not a solution of the equation.

EXAMPLE 6 Distinguishing between Equations and Expressions

Decide whether each of the following is an *equation* or an *expression*.

(a) $2x - 5y$

There is no equals sign, so this is an expression.

(b) $2x = 5y$

Because there is an equals sign with something on either side of it, this is an equation.

Section 1.4 – Examples

EXAMPLE 1 Using Negative Numbers in Applications


Use an integer to express the number in boldface italics in each application.

- (a) The lowest Fahrenheit temperature ever recorded in meteorological records was ***129°*** below zero at Vostok, Antarctica, on July 21, 1983. (*Source: World Almanac and Book of Facts 2006.*)

Use -129 because “below zero” indicates a negative number.

- (b) The shore surrounding the Dead Sea is ***1340*** ft below sea level. (*Source: Microsoft Encarta Encyclopedia.*)

Again, “below sea level” indicates a negative number, -1340 .

 **Now Try Exercises 1 and 3.**

Fractions, introduced in **Section 1.1**, are examples of *rational numbers*.

Rational Numbers

$\{x \mid x \text{ is a quotient of two integers, with denominator not } 0\}$ is the set of **rational numbers**.

(Read the part in the braces as “the set of all numbers x such that x is a quotient of two integers, with denominator not 0.”)

EXAMPLE 3 Determining the Order of Real Numbers

Is it true that $-3 < -1$?

To decide whether the statement is true, locate both numbers, -3 and -1 , on a number line, as shown in Figure 10. Since -3 is to the left of -1 on the number line, -3 is less than -1 . The statement $-3 < -1$ is true.

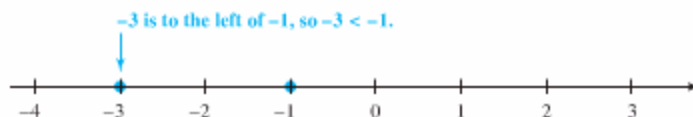


FIGURE 10

EXAMPLE 2 Determining whether a Number Belongs to a Set

List the numbers in the following set that belong to each set of numbers:

$$\left\{-5, -\frac{2}{3}, 0, \sqrt{2}, 3\frac{1}{4}, 5, 5.8\right\}.$$

(a) Natural numbers

The only natural number in the set is 5.

(b) Whole numbers

The whole numbers consist of the natural numbers and 0. So the elements of the set that are whole numbers are 0 and 5.

(c) Integers

The integers in the set are -5 , 0 , and 5 .

(d) Rational numbers

The rational numbers are -5 , $-\frac{2}{3}$, 0 , $3\frac{1}{4}$, 5 , and 5.8 , since each of these numbers *can* be written as the quotient of two integers. For example, $3\frac{1}{4} = \frac{13}{4}$ and $5.8 = \frac{58}{10}$.

(e) Irrational numbers

The only irrational number in the set is $\sqrt{2}$.

(f) Real numbers

All the numbers in the set are real numbers.

EXAMPLE 4 Finding the Absolute Value

Simplify by finding the absolute value.

(a) $|5| = 5$

(b) $|-5| = -(-5) = 5$

(c) $-|5| = -(5) = -5$

(d) $-|-5| = -(5) = -5$

(e) $|8 - 2| = |6| = 6$

(f) $-|8 - 2| = -|6| = -6$

EXAMPLE 5 Interpreting Data

The Producer Price Index (PPI) is the oldest continuous statistical series published by the Bureau of Labor Statistics. It measures the average changes in prices received by producers of all commodities produced in the United States.

The table shows the percent change in the Producer Price Index for selected commodities from 2002 to 2003 and from 2003 to 2004. Use the table to answer each question.

Commodity	Change from 2002 to 2003	Change from 2003 to 2004
Farm products	12.6	10.6
Gasoline	23.3	24.7
Machinery and equipment	-0.8	0.2
Iron and steel	6.5	33.7
Electronic components and accessories	-1.7	-2.2

Source: U.S. Bureau of Labor Statistics.

(a) What commodity in which year represents the greatest percent decrease?

We must find the negative number with the greatest absolute value. The number that satisfies this condition is -2.2 ; the greatest percent decrease was shown by electronic components and accessories from 2003 to 2004.

(b) Which commodity in which year represents the least change?

In this case, we must find the number (either positive, negative, or zero) with the least absolute value. From 2003 to 2004, machinery and equipment showed the least change, an increase of 0.2%.

Section 1.4 – Exercises

For Exercises 19 and 20, see Example 2. List all numbers from each set that are

- (a) natural numbers; (b) whole numbers; (c) integers;
(d) rational numbers; (e) irrational numbers; (f) real numbers.

19. $\left\{-9, -\sqrt{7}, -1\frac{1}{4}, -\frac{3}{5}, 0, \sqrt{5}, 3, 5.9, 7\right\}$

20. $\left\{-5.3, -5, -\sqrt{3}, -1, -\frac{1}{9}, 0, 1.2, 1.8, 3, \sqrt{11}\right\}$

Concept Check Give three numbers between -6 and 6 that satisfy each given condition.

69. Positive real numbers but not integers 70. Real numbers but not positive numbers
71. Real numbers but not whole numbers 72. Rational numbers but not integers
73. Real numbers but not rational numbers 74. Rational numbers but not negative numbers

Section 1.5 – Examples

EXAMPLE 4 Adding Mentally

Check each answer, trying to work the addition mentally. If you have trouble, use a number line.

(a) $7 + (-4) = 3$

(b) $-8 + 12 = 4$

(c) $-\frac{1}{2} + \frac{1}{8} = -\frac{4}{8} + \frac{1}{8} = -\frac{3}{8}$

Remember to find a common denominator.

(d) $\frac{5}{6} + \left(-1\frac{1}{3}\right) = \frac{5}{6} + \left(-\frac{4}{3}\right) = \frac{5}{6} + \left(-\frac{8}{6}\right) = -\frac{3}{6} = -\frac{1}{2}$

(e) $-4.6 + 8.1 = 3.5$

(f) $-16 + 16 = 0$

(g) $42 + (-42) = 0$

EXAMPLE 6 Adding and Subtracting with Grouping Symbols

Perform each indicated operation.

Start here.

$$\begin{aligned}
 \text{(a)} \quad & -6 - [2 - (8 + 3)] \\
 & = -6 - [2 - 11] && \text{Add.} \\
 & = -6 - [2 + (-11)] && \text{Definition of subtraction} \\
 & = -6 - [-9] && \text{Add.} \\
 & = -6 + (9) && \text{Definition of subtraction} \\
 & = 3 && \text{Add.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & 5 + [(-3 - 2) - (4 - 1)] \\
 & = 5 + [(-3 + (-2)) - 3] \\
 & = 5 + [(-5) - 3] \\
 & = 5 + [(-5) + (-3)] \\
 & = 5 + [-8] \\
 & = -3
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad & \frac{2}{3} - \left[\frac{1}{12} - \left(-\frac{1}{4} \right) \right] \\
 & = \frac{8}{12} - \left[\frac{1}{12} - \left(-\frac{3}{12} \right) \right] && \text{Find a common denominator.} \\
 & = \frac{8}{12} - \left[\frac{1}{12} + \frac{3}{12} \right] && \text{Definition of subtraction} \\
 & = \frac{8}{12} - \frac{4}{12} && \text{Add.} \\
 & = \frac{4}{12} && \text{Subtract.} \\
 & = \frac{1}{3} && \text{Lowest terms}
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & |4 - 7| + 2|6 - 3| \\
 & = |-3| + 2|3| && \text{Work within absolute value bars.} \\
 & = 3 + 2 \cdot 3 && \text{Evaluate absolute values.} \\
 & = 3 + 6 && \text{Multiply.} \\
 & = 9 && \text{Add.}
 \end{aligned}$$

Be careful!
Multiply first.

✓ Now Try Exercises 65.

EXAMPLE 7 Interpreting Words and Phrases Involving Addition

Write a numerical expression for each phrase and simplify the expression.

- (a) The
- sum of**
- -8
- and
- 4
- and
- 6

$$-8 + 4 + 6$$

To simplify, add in order from left to right, to obtain

$$-4 + 6, \text{ or } 2.$$

- (b)
- 3 more than**
- -5
- ,
- increased by**
- 12

$$-5 + 3 + 12 \text{ simplifies to } -2 + 12, \text{ or } 10.$$

EXAMPLE 8 Interpreting Words and Phrases Involving Subtraction

Write a numerical expression for each phrase and simplify the expression.

- (a) The **difference between** -8 and 5

When “difference between” is used, write the numbers in the order they are given.* The expression is

$$-8 - 5, \text{ which simplifies to } -8 + (-5), \text{ or } -13.$$

- (b) 4 **subtracted from** the sum of 8 and -3

Here, addition is also used, as indicated by the word *sum*. First, add 8 and -3 . Next, subtract 4 from this sum. The expression is

$$[8 + (-3)] - 4, \text{ which simplifies to } 5 - 4, \text{ or } 1.$$

- (c) 4 **less than** -6

Here, 4 must be taken *from* -6 , so write -6 first.

Be careful
with order.

$$-6 - 4 \text{ simplifies to } -6 + (-4), \text{ or } -10.$$

Notice that “ 4 less than -6 ” differs from “ 4 is less than -6 .” The second of these is symbolized $4 < -6$ (which is a false statement).

- (d) 8 , **decreased by** 5 **less than** 12

First, write “ 5 less than 12 ” as $12 - 5$. Next, subtract $12 - 5$ from 8 .

$$8 - (12 - 5) \text{ simplifies to } 8 - 7, \text{ or } 1.$$

EXAMPLE 9 Solving a Problem Involving Subtraction

The record-high temperature in the United States is 134° Fahrenheit, recorded at Death Valley, California, in 1913. The record low is -80°F , at Prospect Creek, Alaska, in 1971. See Figure 16. What is the difference between these highest and lowest temperatures? (*Source: National Climatic Data Center.*)

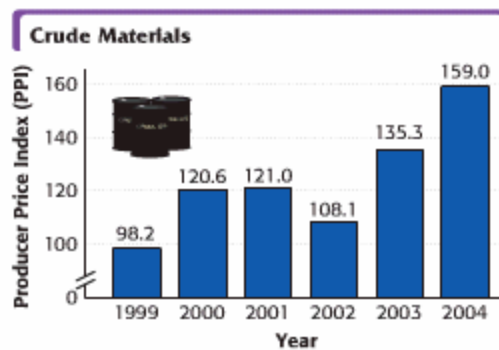
We must subtract the lowest temperature from the highest temperature.

$$\begin{aligned} 134 - (-80) &= 134 + 80 && \text{Definition of subtraction} \\ &= 214 && \text{Add.} \end{aligned}$$

The difference between the two temperatures is 214°F .

EXAMPLE 10 Using a Signed Number to Interpret Data

The bar graph in Figure 17 on the next page gives the Producer Price Index (PPI) for crude materials between 1999 and 2004.



Source: U.S. Bureau of Labor Statistics.

FIGURE 17

(a) Use a signed number to represent the change in the PPI from 2003 to 2004.

To find this change, we start with the index number for 2004 and then subtract from it the index number for 2003.

$$\underbrace{159.0}_{\text{2004 index}} - \underbrace{135.3}_{\text{2003 index}} = \underbrace{+23.7}_{\text{A positive number indicates an increase.}}$$

(b) Use a signed number to represent the change in the PPI from 2001 to 2002.

$$\underbrace{108.1}_{\text{2002 index}} - \underbrace{121.0}_{\text{2001 index}} = 108.1 + (-121.0) = \underbrace{-12.9}_{\text{A negative number indicates a decrease.}}$$

Section 1.5 – Exercises

Write a numerical expression for each phrase and simplify. See Examples 7 and 8.

- | | |
|--|---|
| 83. The sum of -5 and 12 and 6 | 84. The sum of -3 and 5 and -12 |
| 85. 14 added to the sum of -19 and -4 | 86. -2 added to the sum of -18 and 11 |
| 87. The sum of -4 and -10 , increased by 12 | 88. The sum of -7 and -13 , increased by 14 |
| 89. $\frac{2}{7}$ more than the sum of $\frac{5}{7}$ and $-\frac{9}{7}$ | 90. 1.85 more than the sum of -1.25 and -4.75 |
| 91. The difference between 4 and -8 | 92. The difference between 7 and -14 |
| 93. 8 less than -2 | 94. 9 less than -13 |
| 95. The sum of 9 and -4 , decreased by 7 | 96. The sum of 12 and -7 , decreased by 14 |
| 97. 12 less than the difference between 8 and -5 | 98. 19 less than the difference between 9 and -2 |

The two tables show the heights of some selected mountains and the depths of some selected trenches. Use the information given to answer Exercises 103–108.

Mountain	Height (in feet)
Foraker	17,400
Wilson	14,246
Pikes Peak	14,110

Trench	Depth (in feet, as a negative number)
Philippine	-32,995
Cayman	-24,721
Java	-23,376

Source: *World Almanac and Book of Facts 2006*.

103. What is the difference between the height of Mt. Foraker and the depth of the Philippine Trench?
 104. What is the difference between the height of Pikes Peak and the depth of the Java Trench?
 105. How much deeper is the Cayman Trench than the Java Trench?
 106. How much deeper is the Philippine Trench than the Cayman Trench?
-
109. On the basis of census population projections for 2020, New York will lose 5 seats in the U.S. House of Representatives, Pennsylvania will lose 4 seats, and Ohio will lose 3. Write a signed number that represents the total number of seats these three states are projected to lose. (Source: Population Reference Bureau.)