

Section 1.1 - Examples

EXAMPLE 2 Writing Fractions in Lowest Terms

Write each fraction in lowest terms.

$$(a) \frac{10}{15} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{2 \cdot 1}{3 \cdot 1} = \frac{2}{3}$$

The factored form shows that 5 is the greatest common factor of 10 and 15. Dividing both numerator and denominator by 5 gives $\frac{10}{15}$ in lowest terms as $\frac{2}{3}$.

$$(b) \frac{15}{45} = \frac{3 \cdot 5}{3 \cdot 3 \cdot 5} = \frac{1 \cdot 1}{3 \cdot 1 \cdot 1} = \frac{1}{3}$$

The factored form shows that 3 and 5 are the prime factors of both 15 and 45. Dividing both 15 and 45 by $3 \cdot 5 = 15$ gives $\frac{15}{45}$ in lowest terms as $\frac{1}{3}$.

✓ Now Try Exercises 25 and 27.

We can simplify this process by finding the greatest common factor in the numerator and denominator by inspection. For instance, in Example 2(b), we can use 15 rather than $3 \cdot 5$.

$$\frac{15}{45} = \frac{15}{3 \cdot 15} = \frac{1}{3 \cdot 1} = \frac{1}{3}$$

Remember to write 1 in the numerator.

▶ **CAUTION** When writing fractions like $\frac{15}{45}$ from Example 2(b) in lowest terms, be sure to include the factor 1 in the numerator or an error may result.

OBJECTIVE 3 Multiply and divide fractions.

Multiplying Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.

That is, to multiply two fractions, multiply their numerators and then multiply their denominators.



EXAMPLE 3 Multiplying Fractions

Find each product, and write it in lowest terms.

$$(a) \frac{3}{8} \cdot \frac{4}{9} = \frac{3 \cdot 4}{8 \cdot 9}$$

$$= \frac{3 \cdot 4}{2 \cdot 4 \cdot 3 \cdot 3}$$

$$= \frac{1}{2 \cdot 3}$$

$$= \frac{1}{6}$$

Multiply numerators.

Multiply denominators.

Factor the denominator.

Divide numerator and denominator by 3 · 4, or 12.

Lowest terms

Remember to write 1 in the numerator.

Think: $4 \cdot 5 = 20$, and
 $20 + 1 = 21$, so $5\frac{1}{4} = \frac{21}{4}$.

$$(b) 2\frac{1}{3} \cdot 5\frac{1}{4} = \frac{7}{3} \cdot \frac{21}{4}$$

Write as improper fractions.

$$= \frac{7 \cdot 21}{3 \cdot 4}$$

Multiply numerators.
 Multiply denominators.

Think: $3 \cdot 2 = 6$,
 and $6 + 1 = 7$,
 so $2\frac{1}{3} = \frac{7}{3}$.

$$= \frac{7 \cdot 3 \cdot 7}{3 \cdot 4}$$

Factor the numerator.

$$= \frac{49}{4}, \text{ or } 12\frac{1}{4}$$

Write in lowest terms
 and as a mixed number.

✓ Now Try Exercises 35 and 41.

NOTE Some students prefer to divide out any common factors before multiplying. For example,

$$\frac{3}{8} \cdot \frac{4}{9} = \frac{3}{2 \cdot 4} \cdot \frac{4}{3 \cdot 3}$$

Example 3(a)

$$= \frac{1}{2 \cdot 3} = \frac{1}{6}$$

The same answer results.

Two fractions are **reciprocals** of each other if their product is 1. For example,

$$\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1,$$

so $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals, as are $\frac{1}{6}$ and 6 (since $6 = \frac{6}{1}$). Because division is the opposite (or inverse) of multiplication, we use reciprocals to divide fractions.

Dividing Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

That is, to divide by a fraction, multiply by its reciprocal.

We will explain why this method works in **Chapter 7**. However, as an example, we know that $20 \div 10 = 2$ and $20 \cdot \frac{1}{10} = 2$. The answer to a division problem is called a **quotient**. For example, the quotient of 20 and 10 is 2.

EXAMPLE 4 Dividing Fractions

Find each quotient, and write it in lowest terms.

$$(a) \frac{3}{4} \div \frac{8}{5} = \frac{3}{4} \cdot \frac{5}{8} = \frac{3 \cdot 5}{4 \cdot 8} = \frac{15}{32}$$

Multiply by the reciprocal of the second fraction.

$$(b) \frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \cdot \frac{8}{5} = \frac{3 \cdot 8}{4 \cdot 5} = \frac{3 \cdot 4 \cdot 2}{4 \cdot 5} = \frac{6}{5}, \text{ or } 1\frac{1}{5}$$

Remember to write in lowest terms.

$$(c) \frac{5}{8} \div 10 = \frac{5}{8} \div \frac{10}{1} = \frac{5}{8} \cdot \frac{1}{10} = \frac{5 \cdot 1}{8 \cdot 10} = \frac{5 \cdot 1}{8 \cdot 5 \cdot 2} = \frac{1}{16}$$

Write 10 as $\frac{10}{1}$.

$$(d) 1\frac{2}{3} \div 4\frac{1}{2} = \frac{5}{3} \div \frac{9}{2} \quad \text{Write as improper fractions.}$$

$$= \frac{5}{3} \cdot \frac{2}{9} \quad \text{Multiply by the reciprocal of the second fraction.}$$

$$= \frac{10}{27} \quad \text{Multiply numerators.}$$

$$\quad \text{Multiply denominators.}$$

✓ Now Try Exercises 43, 45, and 49.

OBJECTIVE 4 Add and subtract fractions. The result of adding two numbers is called the **sum** of the numbers. For example, $2 + 3 = 5$, so 5 is the sum of 2 and 3.

Adding Fractions

$$\text{If } \frac{a}{b} \text{ and } \frac{c}{b} \text{ are fractions, then } \frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}.$$

That is, to find the sum of two fractions having the *same* denominator, add the numerators and keep the *same* denominator.



EXAMPLE 5 Adding Fractions with the Same Denominator

Find each sum, and write it in lowest terms.

$$(a) \frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7} \quad \text{Add numerators; keep the same denominator.}$$

$$(b) \frac{2}{10} + \frac{3}{10} = \frac{2+3}{10} = \frac{5}{10} = \frac{1}{2} \quad \text{Write in lowest terms.}$$

✓ Now Try Exercise 55.

If the fractions to be added do not have the same denominators, we can still use the preceding method, but only *after* we rewrite the fractions with a common denominator. For example, to rewrite $\frac{3}{4}$ as a fraction with denominator 32, that is,

$$\frac{3}{4} = \frac{?}{32},$$

we must find the number that can be multiplied by 4 to give 32. Since $4 \cdot 8 = 32$, we use the number 8.

$$\frac{3}{4} = \frac{3 \cdot 8}{4 \cdot 8} = \frac{24}{32} \quad \text{Multiply numerator and denominator by 8.}$$

Finding the Least Common Denominator

To add or subtract fractions with different denominators, find the **least common denominator (LCD)** as follows:

Step 1 Factor each denominator.

Step 2 For the LCD, use every factor that appears in any factored form. If a factor is repeated, use the largest number of repeats in the LCD.

EXAMPLE 6 Adding Fractions with Different Denominators

Find each sum, and write it in lowest terms.

(a) $\frac{4}{15} + \frac{5}{9}$

To find the least common denominator, first factor both denominators.

$$15 = 5 \cdot 3 \quad \text{and} \quad 9 = 3 \cdot 3$$

Since 5 and 3 appear as factors, and 3 is a factor of 9 twice, the LCD is

$$\begin{array}{c} 15 \quad 9 \\ \diagdown \quad \diagup \\ 5 \cdot 3 \cdot 3, \quad \text{or} \quad 45. \end{array}$$

Write each fraction with 45 as denominator.

$$\frac{4}{15} = \frac{4 \cdot 3}{15 \cdot 3} = \frac{12}{45} \quad \text{and} \quad \frac{5}{9} = \frac{5 \cdot 5}{9 \cdot 5} = \frac{25}{45}$$

Now add the two equivalent fractions.

$$\frac{4}{15} + \frac{5}{9} = \frac{12}{45} + \frac{25}{45} = \frac{37}{45}$$

(b) $3\frac{1}{2} + 2\frac{3}{4}$

We add mixed numbers by using either of two methods.

Method 1 Write the mixed numbers as improper fractions.

$$\begin{aligned} 3\frac{1}{2} + 2\frac{3}{4} &= \frac{7}{2} + \frac{11}{4} && \text{Change to improper fractions.} \\ &= \frac{14}{4} + \frac{11}{4} && \text{Get a common denominator.} \\ &= \frac{25}{4}, \quad \text{or} \quad 6\frac{1}{4} && \text{Add; write as a mixed number.} \end{aligned}$$

Method 2 Write $3\frac{1}{2}$ as $3\frac{2}{4}$. Then add vertically.

$$\begin{array}{r} 3\frac{1}{2} = 3\frac{2}{4} \\ + 2\frac{3}{4} = 2\frac{3}{4} \\ \hline 5\frac{5}{4} = 5 + 1\frac{1}{4} = 6\frac{1}{4}, \quad \text{or} \quad \frac{25}{4} \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Add the whole numbers and} \\ \text{the fractions separately.} \end{array}$$

Subtracting Fractions

If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions, then $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$.

That is, to find the difference between two fractions having the *same* denominator, subtract the numerators and keep the *same* denominator.

EXAMPLE 7 Subtracting Fractions

Find each difference, and write it in lowest terms.

(a) $\frac{15}{8} - \frac{3}{8} = \frac{15 - 3}{8}$ Subtract numerators; keep the same denominator.
 $= \frac{12}{8} = \frac{3}{2}$, or $1\frac{1}{2}$ Write in lowest terms and as a mixed number.

(b) $\frac{7}{18} - \frac{4}{15}$
Here, $18 = 2 \cdot 3 \cdot 3$ and $15 = 3 \cdot 5$, so the LCD is $2 \cdot 3 \cdot 3 \cdot 5 = 90$.
 $\frac{7}{18} - \frac{4}{15} = \frac{7 \cdot 5}{2 \cdot 3 \cdot 3 \cdot 5} - \frac{4 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 3 \cdot 5} = \frac{35}{90} - \frac{24}{90} = \frac{11}{90}$

(c) $\frac{15}{32} - \frac{11}{45}$
Since $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ and $45 = 3 \cdot 3 \cdot 5$, there are no common factors, and the LCD is $32 \cdot 45 = 1440$.

$$\begin{aligned}\frac{15}{32} - \frac{11}{45} &= \frac{15 \cdot 45}{32 \cdot 45} - \frac{11 \cdot 32}{45 \cdot 32} && \text{Get a common denominator.} \\ &= \frac{675}{1440} - \frac{352}{1440} \\ &= \frac{323}{1440} && \text{Subtract numerators; keep the common denominator.}\end{aligned}$$

(d) $4\frac{1}{2} - 1\frac{3}{4} = \frac{9}{2} - \frac{7}{4}$ Change to improper fractions.
 $= \frac{18}{4} - \frac{7}{4}$ Get a common denominator.
 $= \frac{11}{4}$, or $2\frac{3}{4}$ Subtract; write as a mixed number.

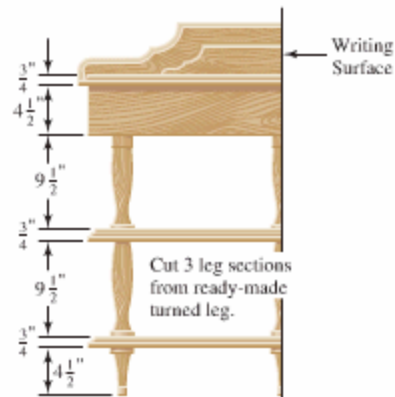


FIGURE 1

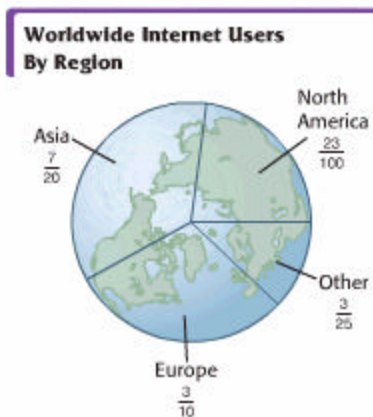
$$\begin{array}{r}
 \frac{3}{4} \rightarrow \frac{3}{4} \\
 4\frac{1}{2} = 4\frac{2}{4} \\
 9\frac{1}{2} = 9\frac{2}{4} \\
 \frac{3}{4} \rightarrow \frac{3}{4} \\
 9\frac{1}{2} = 9\frac{2}{4} \\
 \frac{3}{4} \rightarrow \frac{3}{4} \\
 + 4\frac{1}{2} = 4\frac{2}{4} \\
 \hline
 26\frac{17}{4}
 \end{array}$$

Since $\frac{17}{4} = 4\frac{1}{4}$, $26\frac{17}{4} = 26 + 4\frac{1}{4} = 30\frac{1}{4}$. The height is $30\frac{1}{4}$ in.

Now Try Exercise 77.

EXAMPLE 9 Using a Circle Graph to Interpret Information

In November 2005, there were about 970 million Internet users worldwide. The circle graph in Figure 2 shows the approximate fractions of these users living in various regions of the world.



Source: www.internetworldstats.com

FIGURE 2



(a) Which region had the largest share of Internet users in November 2005? What was that share?

In the circle graph, the sector for Asia is the largest, so Asia had the largest share of Internet users, $\frac{7}{20}$.

(b) Estimate the number of Internet users in North America in November 2005.

A share of $\frac{23}{100}$ can be rounded to $\frac{25}{100}$, or $\frac{1}{4}$, and the total number of Internet users, 970 million, can be rounded to 1000 million (1 billion). We multiply $\frac{1}{4}$ by 1000. The number of Internet users in North America would be about

$$\frac{1}{4}(1000) = 250 \text{ million.}$$

(c) How many actual Internet users were there in North America in November 2005?

To find the answer, we multiply the actual fraction from the graph for North America, $\frac{23}{100}$, by the number of users, 970 million:

$$\frac{23}{100}(970) = \frac{23}{100} \cdot \frac{970}{1} = \frac{22,310}{100} = 223\frac{1}{10}.$$

Thus, $223\frac{1}{10}$ million, or 223,100,000, people in North America used the Internet. This number is reasonable, given our estimate in part (b).

Section 1.1 – Exercises

Write each fraction in lowest terms. See Example 2.

23. $\frac{8}{16}$

24. $\frac{4}{12}$

 25. $\frac{15}{18}$

26. $\frac{16}{20}$

Find each product or quotient, and write it in lowest terms. See Examples 3 and 4.

33. $\frac{4}{5} \cdot \frac{6}{7}$

34. $\frac{5}{9} \cdot \frac{10}{7}$

 35. $\frac{1}{10} \cdot \frac{12}{5}$

36. $\frac{6}{11} \cdot \frac{2}{3}$

37. $\frac{15}{4} \cdot \frac{8}{25}$

38. $\frac{4}{7} \cdot \frac{21}{8}$

39. $3\frac{1}{4} \cdot 1\frac{2}{3}$

40. $2\frac{2}{3} \cdot 5\frac{4}{5}$

41. $2\frac{3}{8} \cdot 3\frac{1}{5}$

42. $3\frac{3}{5} \cdot 7\frac{1}{6}$

 43. $\frac{5}{4} \div \frac{3}{8}$

44. $\frac{7}{6} \div \frac{9}{10}$


45. $\frac{32}{5} \div \frac{8}{15}$

46. $\frac{24}{7} \div \frac{6}{21}$

47. $\frac{3}{4} \div 12$

48. $\frac{2}{5} \div 30$

Find each sum or difference, and write it in lowest terms. See Examples 5–7.

 55. $\frac{7}{12} + \frac{1}{12}$

56. $\frac{3}{16} + \frac{5}{16}$

 57. $\frac{5}{9} + \frac{1}{3}$


58. $\frac{4}{15} + \frac{1}{5}$

59. $3\frac{1}{8} + 2\frac{1}{4}$

60. $4\frac{2}{3} + 2\frac{1}{6}$

61. $3\frac{1}{4} + 1\frac{4}{5}$

62. $5\frac{3}{4} + 3\frac{1}{3}$

 63. $\frac{13}{15} - \frac{3}{15}$

64. $\frac{11}{12} - \frac{3}{12}$

65. $\frac{7}{12} - \frac{1}{9}$

66. $\frac{11}{16} - \frac{1}{12}$

67. $4\frac{3}{4} - 1\frac{2}{5}$

68. $8\frac{4}{5} - 7\frac{4}{9}$

69. $6\frac{1}{4} - 5\frac{1}{3}$

70. $5\frac{1}{3} - 2\frac{1}{2}$

Section 1.2 – Examples

EXAMPLE 1 Evaluating Exponential Expressions

Find the value of each exponential expression.

(a) $5^2 = \underline{5 \cdot 5} = 25$

↑ 5 is used as a factor 2 times.

Read 5^2 as “5 squared” or “the square of 5.”

(b) $6^3 = \underline{6 \cdot 6 \cdot 6} = 216$

↑ 6 is used as a factor 3 times.

Read 6^3 as “6 cubed” or “the cube of 6.”

(c) $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ 2 is used as a factor 5 times.

Read 2^5 as “2 to the fifth power.”

(d) $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$ $\frac{2}{3}$ is used as a factor 3 times.

Order of Operations

If grouping symbols are present, simplify within them, innermost first (and above and below fraction bars separately), in the following order:

Step 1 Apply all exponents.

Step 2 Do any multiplications or divisions in the order in which they occur, working from left to right.

Step 3 Do any additions or subtractions in the order in which they occur, working from left to right.

If no grouping symbols are present, start with Step 1.

When no operation is indicated, as in $3(7)$ or $(-5)(-4)$, multiplication is understood.



EXAMPLE 2 Using the Rules for Order of Operations

Find the value of each expression.

(a) $4 + 5 \cdot 6$

Use the rules for order of operations given in the box.

$$4 + 5 \cdot 6$$

Be careful!
Multiply first.

$$= 4 + 30$$

Multiply.

$$= 34$$

Add.

(b) $9(6 + 11)$

$= 9(17)$ Work inside parentheses.

$= 153$ Multiply.

(c) $6 \cdot 8 + 5 \cdot 2$

$= 48 + 10$ Multiply, working from left to right.

$= 58$ Add.

Start here.

(d) $2(5 + 6) + 7 \cdot 3$

$= 2(11) + 7 \cdot 3$ Work inside parentheses.

$= 22 + 21$ Multiply.

$= 43$ Add.

$2^3 = 2 \cdot 2 \cdot 2$, not $2 \cdot 3$.

(e) $9 - 2^3 + 5$

$= 9 - 2 \cdot 2 \cdot 2 + 5$ Apply the exponent.

$= 9 - 8 + 5$ Multiply.

$= 1 + 5$ Subtract.

$= 6$ Add.

(f) $72 \div 2 \cdot 3 + 4 \cdot 2^3 - 3^3$

Think: $3^3 = 3 \cdot 3 \cdot 3$

$= 72 \div 2 \cdot 3 + 4 \cdot 8 - 27$ Apply the exponents.

$= 36 \cdot 3 + 4 \cdot 8 - 27$ Divide.

$= 108 + 32 - 27$ Multiply.

$= 140 - 27$ Add.

$= 113$ Subtract.

Multiplications and divisions are done from left to right *as they appear*; then additions and subtractions are done from left to right, *as they appear*.

✓ Now Try Exercises 25, 31, 39, and 41.

OBJECTIVE 3 Use more than one grouping symbol. An expression with double (or nested) parentheses, such as $2(8 + 3(6 + 5))$, can be confusing. For clarity, we often use brackets, $[]$, in place of one pair of parentheses.

EXAMPLE 3 Using Brackets and Fraction Bars as Grouping Symbols

Simplify each expression.

(a) $2[8 + 3(6 + 5)]$

Start here.

$= 2[8 + 3(11)]$ Add inside parentheses.

$= 2[8 + 33]$ Multiply inside brackets.

$= 2[41]$ Add inside brackets.

$= 82$ Multiply.

(b) $\frac{4(5 + 3) + 3}{2(3) - 1}$

The expression can be written as the quotient

$$[4(5 + 3) + 3] \div [2(3) - 1],$$

which shows that the fraction bar groups the numerator and denominator separately. Simplify both numerator and denominator, and then divide if possible.

$$\begin{aligned} & \frac{4(5 + 3) + 3}{2(3) - 1} \\ &= \frac{4(8) + 3}{2(3) - 1} && \text{Work inside parentheses.} \\ &= \frac{32 + 3}{6 - 1} && \text{Multiply.} \\ &= \frac{35}{5} && \text{Add and subtract.} \\ &= 7 && \text{Divide.} \end{aligned}$$



EXAMPLE 5 Translating from Words to Symbols

Write each word statement in symbols.

- (a) Twelve **equals** ten **plus** two.

$$12 = 10 + 2$$

- (c) Fifteen **is not equal to** eighteen.

$$15 \neq 18$$

- (e) Thirteen **is less than or equal to** forty.

$$13 \leq 40$$

- (b) Nine **is less than** ten.

$$9 < 10$$

- (d) Seven **is greater than** four.

$$7 > 4$$

- (f) Eleven **is greater than or equal to** eleven.

$$11 \geq 11$$