Section 1.6 – Examples

EXAMPLE 5 **Using the Rules for Order of Operations**

Perform each indicated operation.

(a) $-9(2) - (-3)(2)$

$= -18 - (-6)$ Multiply.

$= -18 + 6$ Definition of subtraction

$= -12$ Add.

(b) $-6(-2) - 3(-4)$

$= 12 - (-12)$

$= 12 + 12$

$= 24$

(c) $-5(-2 - 3)$

$= -5(-5)$

$= 25$

(d) $\dfrac{5(-2) - 3(4)}{2(1 - 6)}$

$= \dfrac{-10 - 12}{2(-5)}$ Simplify the numerator and denominator separately.

$= \dfrac{-22}{-10} = \dfrac{11}{5}$ Remember to write in lowest terms.

✔ Now Try Exerci

EXAMPLE 6 **Evaluating Expressions for Numerical Values**

Evaluate each expression, given that $x = -1$, $y = -2$, and $m = -3$.

(a) $(3x + 4y)(-2m)$

$= [3(-1) + 4(-2)][-2(-3)]$ Substitute the given values for the variables.

Use parentheses around substituted negative values to avoid errors.

$= [-3 + (-8)][6]$ Multiply.

$= [-11]6$ Add inside the brackets.

$= -66$ Multiply.

(b) $2x^2 - 3y^2$

$= 2(-1)^2 - 3(-2)^2$ Substitute.

$= 2(1) - 3(4)$ Apply the exponents.

$= 2 - 12$ Multiply.

$= -10$ Subtract.

(c) $\dfrac{4y^2 + x}{m}$

$= \dfrac{4(-2)^2 + (-1)}{-3}$ Substitute.

$= \dfrac{4(4) + (-1)}{-3}$ Apply the exponent.

$= \dfrac{16 + (-1)}{-3}$ Multiply.

$= \dfrac{15}{-3}$, or -5 Add, then divide.

EXAMPLE 7 **Interpreting Words and Phrases Involving Multiplication**

Write a numerical expression for each phrase and simplify the expression.

(a) The product of 12 and the sum of 3 and -6

Here, 12 is multiplied by "the sum of 3 and -6." The expression is

$$12[3 + (-6)], \quad \text{which simplifies to} \quad 12[-3], \quad \text{or} \quad -36.$$

(b) Twice the difference between 8 and -4

$$2[8 - (-4)] \quad \text{simplifies to} \quad 2[12], \quad \text{or} \quad 24.$$

(c) Two-thirds of the sum of -5 and -3

$$\frac{2}{3}[-5 + (-3)] \quad \text{simplifies to} \quad \frac{2}{3}[-8], \quad \text{or} \quad -\frac{16}{3}.$$

(d) 15% of the difference between 14 and -2

$$0.15[14 - (-2)] \quad \text{simplifies to} \quad 0.15[16], \quad \text{or} \quad 2.4.$$

Remember that 15% = 0.15.

(e) Double the product of 3 and 4

$$2 \cdot (3 \cdot 4) \quad \text{simplifies to} \quad 2(12), \quad \text{or} \quad 24.$$

EXAMPLE 8 **Interpreting Words and Phrases Involving Division**

Write a numerical expression for each phrase and simplify the expression.

(a) The quotient of 14 and the sum of -9 and 2

"Quotient" indicates division. The number 14 is the numerator and "the sum of -9 and 2" is the denominator. The expression is

$$\frac{14}{-9 + 2}, \quad \text{which simplifies to} \quad \frac{14}{-7}, \quad \text{or} \quad -2.$$

(b) The product of 5 and -6, divided by the difference between -7 and 8

The numerator of the fraction representing the division is found by multiplying 5 and -6. The denominator is found by subtracting -7 and 8. The expression is

$$\frac{5(-6)}{-7 - 8}, \quad \text{which simplifies to} \quad \frac{-30}{-15}, \quad \text{or} \quad 2.$$

EXAMPLE 9 Translating Sentences into Equations

Write each sentence in symbols, using x as the variable. Then guess or use trial and error to find the solution, which comes from the list of integers between -12 and 12, inclusive.

(a) Three times a number is -18.

The word *times* indicates multiplication, and the word *is* translates as the equals sign (=).

$$3x = -18 \qquad 3 \cdot x = 3x$$

Since the integer between -12 and 12, inclusive, that makes this statement true is -6, the solution of the equation is -6.

(b) The sum of a number and 9 is 12.

$$x + 9 = 12$$

Since $3 + 9 = 12$, the solution of this equation is 3.

(c) The difference between a number and 5 is 0.

$$x - 5 = 0$$

Since $5 - 5 = 0$, the solution of this equation is 5.

(d) The quotient of 24 and a number is -2.

$$\frac{24}{x} = -2$$

Here, x must be a negative number, since the numerator is positive and the quotient is negative. Since $\frac{24}{-12} = -2$, the solution is -12.

Summary page 66

Perform each indicated operation.

1. $14 - 3 \cdot 10$

2. $-3(8) - 4(-7)$

3. $(3 - 8)(-2) - 10$

4. $-6(7 - 3)$

5. $7 - (-3)(2 - 10)$

6. $-4[(-2)(6) - 7]$

7. $(-4)(7) - (-5)(2)$

8. $-5[-4 - (-2)(-7)]$

9. $40 - (-2)[8 - 9]$

10. $\dfrac{5(-4)}{-7 - (-2)}$

11. $\dfrac{-3 - (-9 + 1)}{-7 - (-6)}$

12. $\dfrac{5(-8 + 3)}{13(-2) + (-7)(-3)}$

13. $\dfrac{6^2 - 8}{-2(2) + 4(-1)}$

14. $\dfrac{16(-8 + 5)}{15(-3) + (-7 - 4)(-3)}$

15. $\dfrac{9(-6) - 3(8)}{4(-7) + (-2)(-11)}$

16. $\dfrac{2^2 + 4^2}{5^2 - 3^2}$

17. $\dfrac{(2 + 4)^2}{(5 - 3)^2}$

18. $\dfrac{4^3 - 3^3}{-5(-4 + 2)}$

19. $\dfrac{-9(-6) + (-2)(27)}{3(8 - 9)}$

20. $|-4(9)| - |-11|$

21. $\dfrac{6(-10 + 3)}{15(-2) - 3(-9)}$

22. $\dfrac{3^2 - 5^2}{(-9)^2 - 9^2}$

23. $\dfrac{(-10)^2 + 10^2}{-10(5)}$

24. $-\dfrac{3}{4} \div \left(-\dfrac{5}{8}\right)$

25. $\dfrac{1}{2} \div \left(-\dfrac{1}{2}\right)$

26. $\dfrac{8^2 - 12}{(-5)^2 + 2(6)}$

27. $\left[\dfrac{5}{8} - \left(-\dfrac{1}{16}\right)\right] + \dfrac{3}{8}$

28. $\left(\dfrac{1}{2} - \dfrac{1}{3}\right) - \dfrac{5}{6}$

29. $-0.9(-3.7)$

30. $-5.1(-0.2)$

31. $-3^2 - 2^2$

32. $|-2(3) + 4| - |-2|$

33. $40 - (-2)[-5 - 3]$

Evaluate each expression if $x = -2$, $y = 3$, and $a = 4$.

34. $-x + y - 3a$

35. $(x + 6)^3 - y^3$

36. $(x - y) - (a - 2y)$

37. $\left(\dfrac{1}{2}x + \dfrac{2}{3}y\right)\left(-\dfrac{1}{4}a\right)$

38. $\dfrac{2x + 3y}{a - xy}$

39. $\dfrac{x^2 - y^2}{x^2 + y^2}$

40. $-x^2 + 3y$

41. $\left(\dfrac{x}{y}\right)^3$

42. $\left(\dfrac{a}{x}\right)^2$

Section 1.7 – Examples

EXAMPLE 5 **Using the Identity Properties**

These statements are examples of the identity properties.

(a) $-3 + 0 = -3$ Addition **(b)** $1 \cdot \frac{1}{2} = \frac{1}{2}$ Multiplication

✔ **Now Try Exercise 21.**

We use the identity property for multiplication to write fractions in lowest terms and to find common denominators.

EXAMPLE 6 **Using the Identity Property to Simplify Expressions**

Simplify.

(a) $\frac{49}{35} = \frac{7 \cdot 7}{5 \cdot 7}$ Factor.

$= \frac{7}{5} \cdot \frac{7}{7}$ Write as a product.

$= \frac{7}{5} \cdot 1$ Divide.

$= \frac{7}{5}$ Identity property

(b) $\frac{3}{4} + \frac{5}{24} = \frac{3}{4} \cdot 1 + \frac{5}{24}$ Identity property

$= \frac{3}{4} \cdot \frac{6}{6} + \frac{5}{24}$ Use $1 = \frac{6}{6}$ to get a common denominator.

$= \frac{18}{24} + \frac{5}{24}$ Multiply.

$= \frac{23}{24}$ Add.

EXAMPLE 7 **Using the Inverse Properties**

These statements are examples of the inverse properties.

(a) $-\frac{1}{2} + \frac{1}{2} = 0$ **(b)** $4 + (-4) = 0$ **(c)** $-0.75 + \frac{3}{4} = 0$

(d) $\frac{2}{5} \cdot \frac{5}{2} = 1$ **(e)** $-5\left(-\frac{1}{5}\right) = 1$ **(f)** $4(0.25) = 1$

✔ **Now Try Exercise 19.**

EXAMPLE 8 **Using Properties to Simplify an Expression**

Simplify $-2x + 10 + 2x$.

$-2x + 10 + 2x$

$= (-2x + 10) + 2x$ Order of operations

$= [10 + (-2x)] + 2x$ Commutative property

$= 10 + [(-2x) + 2x]$ Associative property

$= 10 + 0$ Inverse property

$= 10$ Identity property

Note that for *any* value of x, $-2x$ and $2x$ are additive inverses; that is why we can use the inverse property in this simplification.

EXAMPLE 9 Using the Distributive Property

Use the distributive property to rewrite each expression.

(a) $5(9 + 6)$

$= 5 \cdot 9 + 5 \cdot 6$ Distributive property

$= 45 + 30$ Multiply.

$= 75$ Add.

Multiply first.

(b) $4(x + 5 + y)$

$= 4x + 4 \cdot 5 + 4y$ Distributive property

$= 4x + 20 + 4y$ Multiply.

(c) $-2(x + 3)$

$= -2x + (-2)(3)$ Distributive property

$= -2x - 6$ Multiply.

(d) $3(k - 9)$

$= 3[k + (-9)]$ Definition of subtraction

$= 3k + 3(-9)$ Distributive property

$= 3k - 27$ Multiply.

(e) $8(3r + 11t + 5z)$

$= 8(3r) + 8(11t) + 8(5z)$ Distributive property

$= (8 \cdot 3)r + (8 \cdot 11)t + (8 \cdot 5)z$ Associative property

$= 24r + 88t + 40z$ Multiply.

(f) $6 \cdot 8 + 6 \cdot 2$

$= 6(8 + 2)$ Distributive property

$= 6(10)$ Add.

$= 60$ Multiply.

(g) $4x - 4m$

$= 4(x - m)$

(h) $6x - 12$

$= 6 \cdot x - 6 \cdot 2$

$= 6(x - 2)$

EXAMPLE 10 Using the Distributive Property to Remove Parentheses

Write each expression without parentheses.

(a) $-(7r - 8)$

$= -1(7r - 8)$ $-a = -1 \cdot a$

$= -1(7r) + (-1)(-8)$ Distributive property

$= -7r + 8$ Multiply.

(b) $-(-9w + 2)$

$= -1(-9w + 2)$

$= -1(-9w) - 1(2)$

$= 9w - 2$

(c) $-(-x - 3y + 6z)$

$= -1(-1x - 3y + 6z)$

$= -1(-1x) - 1(-3y) - 1(6z)$

$= x + 3y - 6z$

Section 1.8 – Examples

EXAMPLE 1 **Simplifying Expressions**

Simplify each expression.

(a) $4x + 8 + 9$ simplifies to $4x + 17$.

(b) $4(3m - 2n)$

$= 4(3m) - 4(2n)$ Distributive property

$= (4 \cdot 3)m - (4 \cdot 2)n$ Associative property

$= 12m - 8n$

(c) $6 + 3(4k + 5)$

$= 6 + 3(4k) + 3(5)$ Distributive property

$= 6 + (3 \cdot 4)k + 3(5)$ Associative property

$= 6 + 12k + 15$ Multiply.

$= 6 + 15 + 12k$ Commutative property

$= 21 + 12k$ Add.

Don't start by adding!

(d) $5 - (2y - 8)$

$= 5 - 1(2y - 8)$ $-a = -1 \cdot a$

$= 5 - 1(2y) - 1(-8)$ Distributive property

$= 5 - 2y + 8$ Multiply.

$= 5 + 8 - 2y$ Commutative property

$= 13 - 2y$ Add.

Be careful with signs.

EXAMPLE 2 **Combining Like Terms**

Combine like terms in each expression.

(a) $-9m + 5m$

$= (-9 + 5)m$

$= -4m$

(b) $6r + 3r + 2r$

$= (6 + 3 + 2)r$

$= 11r$

(c) $4x + x$

$= 4x + 1x$ $x = 1x$

$= (4 + 1)x$

$= 5x$

(d) $16y^2 - 9y^2$

$= (16 - 9)y^2$

$= 7y^2$

(e) $32y + 10y^2$ cannot be combined because $32y$ and $10y^2$ are unlike terms. We cannot use the distributive property here to combine coefficients.

EXAMPLE 3 **Simplifying Expressions Involving Like Terms**

Simplify each expression.

(a) $14y + 2(6 + 3y)$

$= 14y + 2(6) + 2(3y)$ Distributive property

$= 14y + 12 + 6y$ Multiply.

$= 20y + 12$ Combine like terms.

(b) $9k - 6 - 3(2 - 5k)$

Be careful with signs.

$$= 9k - 6 - 3(2) - 3(-5k) \qquad \text{Distributive property}$$
$$= 9k - 6 - 6 + 15k \qquad \text{Multiply.}$$
$$= 24k - 12 \qquad \text{Combine like terms.}$$

(c) $-(2 - r) + 10r$

$$= -1(2 - r) + 10r \qquad -a = -1 \cdot a$$
$$= -1(2) - 1(-r) + 10r \qquad \text{Distributive property}$$
$$= -2 + 1r + 10r \qquad \text{Multiply.}$$
$$= -2 + 11r \qquad \text{Combine like terms.}$$

Be careful with signs.

(d) $100[0.03(x + 4)]$

$$= [(100)(0.03)](x + 4) \qquad \text{Associative property}$$
$$= 3(x + 4) \qquad \text{Multiply.}$$
$$= 3x + 12 \qquad \text{Distributive property}$$

(e) $5(2a - 6) - 3(4a - 9)$

$$= 10a - 30 - 12a + 27 \qquad \text{Distributive property}$$
$$= -2a - 3 \qquad \text{Combine like terms.}$$

(f) $-\frac{2}{3}(x - 6) - \frac{1}{6}x$

$$= -\frac{2}{3}x - \frac{2}{3}(-6) - \frac{1}{6}x \qquad \text{Distributive property}$$
$$= -\frac{2}{3}x + 4 - \frac{1}{6}x \qquad \text{Multiply.}$$
$$= -\frac{4}{6}x + 4 - \frac{1}{6}x \qquad \text{Get a common denominator.}$$
$$= -\frac{5}{6}x + 4 \qquad \text{Combine like terms.}$$

EXAMPLE 4 Translating Words to a Mathematical Expression

Translate to a mathematical expression and simplify: The sum of 9, five times a number, four times the number, and six times the number.

The word "sum" indicates that the terms should be added. Use x to represent the number. Then the phrase translates as

$$9 + 5x + 4x + 6x, \qquad \text{Write with symbols.}$$

which simplifies to

$$9 + 15x. \qquad \text{Combine like terms.}$$

Section 1.8 – Exercises

Simplify each expression. See Examples 1–3.

33. $9y + 8y$

34. $15m + 12m$

35. $-4a - 2a$

36. $-3z - 9z$

37. $12b + b$

38. $30x + x$

39. $2k + 9 + 5k + 6$

40. $2 + 17z + 1 + 2z$

41. $-5y + 3 - 1 + 5 + y - 7$

42. $2k - 7 - 5k + 7k - 3 - k$

43. $-2x + 3 + 4x - 17 + 20$

44. $r - 6 - 12r - 4 + 6r$

45. $16 - 5m - 4m - 2 + 2m$

46. $6 - 3z - 2z - 5 + z - 3z$

47. $-10 + x + 4x - 7 - 4x$

48. $-p + 10p - 3p - 4 - 5p$

49. $1 + 7x + 11x - 1 + 5x$

50. $-r + 2 - 5r + 3 + 4r$

51. $-\frac{4}{3} + 2t + \frac{1}{3}t - 8 - \frac{8}{3}t$

52. $-\frac{5}{6} + 8x + \frac{1}{6}x - 7 - \frac{7}{6}$

53. $6y^2 + 11y^2 - 8y^2$

54. $-9m^3 + 3m^3 - 7m^3$

55. $2p^2 + 3p^2 - 8p^3 - 6p^3$

56. $5y^3 + 6y^3 - 3y^2 - 4y^2$

57. $2(4x + 6) + 3$

58. $4(6y - 9) + 7$

59. $100[0.05(x + 3)]$

60. $100[0.06(x + 5)]$

61. $-4(y - 7) - 6$

62. $-5(t - 13) - 4$

63. $-\frac{4}{3}(y - 12) - \frac{1}{6}y$

64. $-\frac{7}{5}(t - 15) - \frac{1}{2}t$

65. $-5(5y - 9) + 3(3y + 6)$

66. $-3(2t + 4) + 8(2t - 4)$

67. $-3(2r - 3) + 2(5r + 3)$

68. $-4(5y - 7) + 3(2y - 5)$

69. $8(2k - 1) - (4k - 3)$

70. $6(3p - 2) - (5p + 1)$

71. $-2(-3k + 2) - (5k - 6) - 3k - 5$

72. $-2(3r - 4) - (6 - r) + 2r - 5$

75. $-7.5(2y + 4) - 2.9(3y - 6)$

76. $8.4(6t - 6) + 2.4(9 - 3t)$

Translate each phrase into a mathematical expression. Use x as the variable. Combine like terms when possible. See Example 4.

77. Five times a number, added to the sum of the number and three

78. Six times a number, added to the sum of the number and six

79. A number multiplied by -7, subtracted from the sum of 13 and six times the number

80. A number multiplied by 5, subtracted from the sum of 14 and eight times the number

81. Six times a number added to -4, subtracted from twice the sum of three times the number and 4 (*Hint: Twice* means two times.)

82. Nine times a number added to 6, subtracted from triple the sum of 12 and 8 times the number (*Hint: Triple* means three times.)