Chapter 6
Factoring an expression. Solving equations by factoring

Note: Factoring an expression changes it from a *sum* into a *product.*

I Factoring The Greatest Common Factor

This is a direct application of the *distributive property*: $ab+ac=a(b+c)$

Factor as completely as possible:

1. $8x^3+20x-28$
2. $6x^2-12x$
3. $24a^3b-36a^2c^2+48ab^3$
4. $15x^3y^4-5x^2y^3$
5. $-20xy^3+35x^2y-60xy$
6. $9m-12n+8p$
7. $2t^2+8t$
8. x^2-x
9. $a(a+7)+3(a+7)$
10. x^2y-xy^2
11. $z(z-3)-5(z-3)$
12. $\frac{1}{4}d^2-\frac{3}{4}d$

II Factoring by Grouping

We use this method when we usually have four or more terms.

Factor as completely as possible:

1) $x^2+4x+xy+4y$
2) $a^2-ab-3a+3b$
3) $m^2+mn+9m+9n$
4) $x^2-xy-4x+4y$
5) $y^2+5y-7y-35$
6) $1-a+ab-b$
7) $16m^3-4m^2p^2-4mp+p^3$
8) $7z^2+14z-2z-4$
9) $10x^2-15x-2x+3$

III Special Products

Two Terms		Three Terms
Difference of Squares:	$a^2-b^2=(a-b)(a+b)$	Perfect Square Trinomials
Sum of Squares:	a^2+b^2 - not factorable	$a^2+2ab+b^2=(a+b)^2$
Difference of Cubes:	$a^3-b^3=(a-b)(a^2+ab+b^2)$	$a^2-2ab+b^2=(a-b)^2$
Sum of Cubes:	$a^3+b^3=(a+b)(a^2-ab+b^2)$	

Factor as completely as possible:

1) x^2-25
2) $p^2-\frac{1}{9}$
3) w^2+2w+1
4) a^2+4a+4
5) $100x^2+49$
6) m^3-8
7) $64y^3-27$
8) $6x^3+6$
9) $16x^2-40x+25$
10) x^4-1
11) y^8-256
12) x^9+y^9

IV Factoring Trinomials ax^2+bx+c

Case 1 Leading coefficient is1: $a=1$ Factor $x^2+bx+c=(x\ \square\ )(x\ \square\ )$

product = c
sum = b

Factor as completely as possible:

1) x^2+5x+6
2) x^2-5x-6
3) x^2-7x+6
4) x^2-x-6
5) $2a^2+8a+10$
6) $2x+x^2-15$
7) $6t^2-18t+12$
8) $z^2-17z+30$
9) $3w^2-12w-96$
10) $m^2+6m-18$
11) $x^2-3xy+2y^2$
12) $t^2-tz-6z^2$

Case 2 Leading coefficient is not 1: $a\neq 1$ Factor $ax^2+bx+c=ax^2+\square x+\square x+c$ by splitting the middle term bx then using grouping

product = ac
sum = b

Factor as completely as possible:

13) $6x^2+7x-20$
14) $2t^2-7t+3$
15) $6a^2+40a+24$
16) $3x^2-11x-20$
17) $15+6b^2-19b$
18) $25y^2+35y+45$
19) $7x-x^2-10$
20) $14k^3+7k^2-70k$
21) $8a^2+23ab-3b^2$

V Solving Quadratic Equations by the Factoring Method

1. Get the equation into standard from: $ax^2+bx+c=0$
2. Factor the nonzero side of the equation.
3. Use the zero-product rule to set each of the factors equal to zero.
4. Check each of the answers in the original equation.

Solve each equation:

1) $3x(x+1)=2x+2$
2) $(x-2)(x-4)=15$
3) $64=-16t^2+64t+4$
4) $w^2+(w+3)^2=15^2$
5) $x^2+\frac{5}{2}x-\frac{3}{2}=0$
6) $5z^2=5z$
7) $x(2x-3)=-1$
8) $(v+2)(v-5)=8$
9) $x+\frac{10}{x}=7$
10) $t^3+35t=12t^2$
11) $9y^3=49y$
12) $5-(x-1)^2=(x-2)^2$
13) $3x^2\left(x+\frac{1}{2}\right)\left(2x-\frac{1}{3}\right)\left(5x-\frac{1}{2}\right)=0$
14) $(2x)^2=(2x+4)^2-(x+5)^2$
15) $(3x-5)(4x+1)=24$

VI Applications of Quadratic Equations

1) James Bond stands on top of a 240-foot building and throws a film canister upward to a fellow agent in a helicopter 16 feet above the building. The height of the film above the ground t seconds later is given by the formula $h = -16t^2 + 32t + 240$, where h is in feet.

a) What is the initial height of the film canister?
b) How long will it take the film canister to reach the agent in the helicopter?
c) If the agent misses the canister, when will it pass James Bond on the way down?
d) How long will it take the canister to hit the ground?

2) A plastic box has length 4 cm longer than its width. The area of the rectangular top of the box is 77 sq. cm. Find the length and width of the box.

3) As part of a landscaping project, you put in a flower bed measuring 20 feet by 30 feet. To finish off the project, you are putting in a uniform border of pine bark around the outside of the rectangular garden. You have enough pine bark to cover 336 square feet. How wide should the border be?

4) (Section 6.6 - # 28) Two cars left an intersection at the same time. One traveled north. The other traveled 14 mi farther, but to the east. How far apart were they then, if the distance between them was 4 mi more than the distance traveled east?

5) The height, h, of a baseball t seconds after being hit is given by $h = -16t^2 + 64t + 4$. When will the baseball reach a height of 64 ft?

6) (Section 6.6 - # 29) A ladder is leaning against a building. The distance from the bottom of the ladder to the building is 4 ft less than the length of the ladder. How high up the side of the building is the top of the ladder if that distance is 2 ft less than the length of the ladder?

7) A car traveling at 50 ft per second (about 34 mi per hour) can stop in 2.5 seconds after applying the brakes hard. The distance the car travels in feet, t seconds after applying the brakes, is $d = 50t - 10t^2$. How long does it take the car to travel 40 ft?

8) (Section 6.6 - # 21) Find three consecutive odd integers such that the sum of all three is 42 less than the product of the larger two.

9) A stone is thrown downward off a cliff so that the distance it has traveled after t seconds is given by $d = 16t^2 + 40t$, where d is measured in feet. How long will it take the stone to travel 144 feet?

10) One leg of a right triangle is 7 more than twice the other leg. If the square of the hypotenuse is 706, what are the lengths of the two legs?

11) A rancher has 360 yards of fence to enclose a rectangular pasture. If the pasture should be 8000 square yards in area, what should its dimensions be?

12) Find the length of the shorter leg of a right triangle if the longer leg is 12 feet more than the shorter leg and the hypotenuse is 12 feet less than twice the shorter leg.

13) An object has been thrown straight up into the air. The formula $h = vt - 16t^2$ gives the height of the object above the ground after t seconds when it is thrown upward with an initial velocity v. If a cannonball is fired with an upward velocity of 220 feet per second, at what time(s) will it be at a height of 600 ft?

14) Find the dimensions of a rectangle whose width is 2 inches less than half its length and whose area is 160 square inches.

Answers

Section I:

1) $4\left(2x^3+5x-7\right)$; 2) $6x(x-2)$; 3) $12a\left(2a^2b-3ac^2+4b^3\right)$; 4) $5x^2y^3(3xy-1)$;

5) $5xy\left(-4y^2+7x-12\right)$ or $-5xy\left(4y^2-7x+12\right)$; 6) not factorable; 7) $2t(t+4)$; 8) $x(x-1)$; 9) $(a+7)(a+3)$;

10) $xy(x-y)$; 11) $(z-3)(z-5)$; 12) $\frac{1}{4}d(d-3)$.

Section II:

1) $(x+4)(x+y)$; 2) $(a-b)(a-3)$; 3) $(m+n)(m+9)$; 4) $(x-y)(x-4)$; 5) $(y+5)(y-7)$;

6) $(1-a)(1-b)$; 7) $\left(4m-p^2\right)\left(4m^2-p\right)$; 8) $(z+2)(7z-2)$; 9) $(5x-1)(2x-3)$.

Section III:

1) $(x-5)(x+5)$; 2) $\left(p-\frac{1}{3}\right)\left(p+\frac{1}{3}\right)$; 3) $(w+1)^2$; 4) $(a+2)^2$; 5) not factorable;

6) $(m-2)\left(m^2+2m+4\right)$; 7) $(4y-3)\left(16y^2+12y+9\right)$; 8) $6(x+1)\left(x^2-x+1\right)$; 9) $(4x-5)^2$;

10) $(x-1)(x+1)\left(x^2+1\right)$; 11) $(y-2)(y+2)\left(y^2+4\right)\left(y^4+16\right)$; 12) $(x+y)\left(x^2-xy+y^2\right)\left(x^6-x^3y^3+y^6\right)$.

Section IV:

1) $(x+3)(x+2)$; 2) $(x+1)(x-6)$; 3) $(x-6)(x-1)$; 4) $(x+2)(x-3)$; 5) $2\left(a^2+4a+5\right)$;

6) $(x+5)(x-3)$;7) $6(t-2)(t-1)$; 8) $(z-15)(z-2)$; 9) $3(w-8)(w+4)$; 10) not factorable;

11) $(x-y)(x-2y)$; 12) $(t+2z)(t-3z)$; 13) $(3x-4)(2x+5)$; 14) $(2t-1)(t-3)$; 15) $2(3a+2)(a+6)$;

16) $(x-5)(3x+4)$; 17) $(3b-5)(2b-3)$; 18) $5\left(5y^2+7y+9\right)$; 19) $-(x-5)(x-2)$; 20) $7k(2k+5)(k-2)$;

21) $(8a-b)(a+3b)$.

Section V:

1) $\frac{2}{3},-1$; 2) $7,-1$; 3) $\frac{3}{2},\frac{5}{2}$; 4) $9,-12$; 5) $\frac{1}{2},-3$; 6) $0,1$; 7) $\frac{1}{2},1$; 8) $-3,6$; 9) $5,2$; 10) $0,5,7$;

11) $0,-\frac{7}{3},\frac{7}{3}$; 12) $0,3$; 13) $0,-\frac{1}{2},\frac{1}{6},\frac{1}{10}$; 14) 3; 15) $-1,\frac{29}{2}$.

Section VI:

1) b: 1 sec; c: 2 sec; c: 5 sec; 2) 11cm, 7 cm; 3) 3 ft; 4) 34 mi; 5) 1.5 and 2.5 sec; 6) 8 ft; 7) 1 sec; 8) 5, 7, 9; 9) 2 sec; 10) 9 and 25; 11) 80 yd by 200 yd; 12) 36 ft; 13) 3.75 and 10 sec; 14) 8 in by 20 in