REVIEW
Chapter 1 – The Real Number System

> **In class work** : Complete all statements. Solve all exercises.

(Section 1.4)

<u>Definition</u>   **A set** is a collection of objects (elements).

<u>The Set of Natural Numbers</u> $\mathbb{N}$

$$\mathbb{N} = \underline{\hspace{4cm}}$$

<u>The Set of Whole Numbers</u> **W** $\mathbb{N}$ _____ **W**

$$W = \underline{\hspace{4cm}}$$

<u>The Set of Integers</u> $\mathbb{Z}$ $\mathbb{N}$ _____ **W** _____ $\mathbb{Z}$

$$\mathbb{Z} = \underline{\hspace{4cm}}$$

<u>The Set of Rational Numbers</u> $\mathbb{Q}$ $\mathbb{N}$ _____ **W** _____ $\mathbb{Z}$ ____ $\mathbb{Q}$

$$\mathbb{Q} = \underline{\hspace{4cm}}$$

<u>The Set of Irrational Numbers</u>

Examples:

<u>The Set of Real Numbers</u> $\mathbb{R}$ $\mathbb{N}$ _____ **W** _____ $\mathbb{Z}$ ____ $\mathbb{Q}$ __ $\mathbb{R}$

$$\mathbb{R} = \underline{\hspace{4cm}}$$

<u>Exercise #1</u>   Decide whether each statement is true or false:

a) Every natural number is positive.   _____

b) Every whole number is positive.   _____

c) Every integer is a rational number.   _____

<u>Exercise #2</u>   List all numbers from the set $\left\{ -9, -\sqrt{7},\ -1\frac{1}{4},\ -\frac{3}{5},\ 0,\ \sqrt{5},\ 3,\ 5.9,\ 7 \right\}$ that are

a) natural numbers   _____

b) whole numbers but not natural numbers _____

c) odd integers   _____

d) rational numbers   _____

e) irrational numbers   _____

(Section 1.2)

Mathematical Symbols

SYMBOL	MEANING	EXAMPLES
$=$	is equal to	
$\neq$	is not equal to	
$\in$	belongs to (about an element)	
$\notin$	it doesn't belong to	
$<$	is less than	
$\leq$	is less than or equal to	
$>$	is greater than	
$\geq$	is greater than or equal to	

Definition   A number *a* **is less than a number** *b*   ($a < b$) if *a* is to the left of *b* on the number line.

Exercise #3   Write equivalent statements:

         a) $2 \leq 3$ _____

         b) $30 > 5$ _____

         c) $5 > -1 \geq -6$ _____

         d) $-4 < -2$ _____

Exercise #4   Fill in the appropriate ordering symbol: either $<$, $>$, or $=$.

         a)   2 _____ -5

         b)   19 _____ 24 – 10

         c)   4 – 4 _____ $4 \cdot 0$

Exercise #5   Write each word statement in symbols:

         a) Fifteen is equal to five plus ten.   _____

         b) Nine is greater than five minus four.   _____

         c) Sixteen is not equal to nineteen.   _____

         d) Two is less than or equal to three.   _____

(Section 1.7)   **Properties of Real Numbers**

PROPERTIES	ADDITION +	MULTIPLICATION •
COMMUTATIVITY		
ASSOCIATIVITY		
IDENTITY ELEMENT		
INVERSE ELEMENT		
DISTRIBUTIVE PROPERTY		

Exercise #6   Find the opposite and the reciprocal (if any) of each number:

The Number	Its Opposite	Its Reciprocal

The Double Negative Rule

_____

Exercise #7   Identify the property used in each example:

a. $(-23) + (-11) = (-11) + (-23)$ _____

b. $[123(-2)](-3) = 123[(-2)(-3)]$ _____

c. $1 \cdot 23 = 23 \cdot 1$ _____

d. $[(-29) + 17] + 54 = (-29) + [(17 + 54)]$ _____

e. $(-101)(29) = 29(-101)$ _____

f. $100 + 0 = 0 + 100$ _____

(Section 1.4)

The Absolute Value of a Number

Definition (1) **The absolute value of a number** is the distance between the number and the origin on the number line.

$$|a| = dist(a,0)$$

Property $|a| \geq 0$ for any $a \in R$

Definition (2) $$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Exercise #8 Simplify the following:

a) $|-7|$

b) $-(-7)$

c) $-|-7|$

d) $-|-(-7)|$

Exercise #9 Fill in the appropriate ordering symbol: either $<$, $>$, or $=$.

a) $|-3|$ _____ $|-4|$

b) 3 _____ $|-4|$

c) $-|-6|$ _____ $-|-4|$

d) -6 _____ $-(-3)$

e) $-|8|$ _____ $|-9|$

f) $|6-5|$ _____ $|2-6|$

(Sections 1.2, 1.5, 1.6)

Operations with Real Numbers

<u>Adding Real Numbers</u>   Same sign   - Add the absolute values of the numbers.
- The sum has the same sign as the given numbers.

Different signs - Find the difference between the larger absolute value and the smaller.
- The sum has the sign of the number with the larger value.

<u>Subtracting Real Numbers</u> $a - b = a + (-b)$

<u>Multiplying Real Numbers</u>

<u>Dividing Real Numbers</u> $\dfrac{a}{b} = a \cdot \dfrac{1}{b}$

<u>Exponent</u> $a^n = $ _____

a is called _____

n is called _____

<u>Exercise #10</u> Simplify the following:

a) 5^2

b) $(-5)^2$

c) -5^2

d) $-(-5)^2$

e) 2^3

f) $(-2)^3$

g) -2^3

h) $-(-2)^3$

i) $\left(\dfrac{1}{3}\right)^2$

j) $\left(-\dfrac{3}{4}\right)^3$

k) $-\left(-\dfrac{1}{2}\right)^4$

<u>Order of Operations</u> If grouping symbols are present, simplify within them, innermost first, in the following order:

Step 1 _____

Step 2 _____

Step 3 _____

Exercise #11 Simplify the following:

a) $\left|7\cdot 2-8^2\right|$

b) $(-5)^2 - 3^2 + \left|10-2\cdot 3\right|$

c) $-18 \div (-3)^2 + \left|-8\right| - \left|-4\right|$

d) $\dfrac{(-4)^2 - \left|1-2^3\right|}{-(-2)^3 + (-1)^{125}}$

e) $\dfrac{\left|-8-4\right| \div \left(2-2^2\right)}{-18 \div (-3)^2 + \left|-8\right| - \left|-4\right|}$

f) $238\cdot 0 - 230 \div 10 + 999 \div 9 - 31\cdot 100$

g) $-2(-5)^2 + 10 \div (2) - (-3)^2 (2) + 4^2 \div (-2)$

h) $(4-7)(20-21)^3 - 2[-10(-3) + 2(-1-3)]$

i) $-2(-1)(-7)(-6) + (-2)(-1-7) - 3(2-5)$

j) $\left|2-5\right| + \left|7+10\right| - \left|9-12\right| + \left|0-9\right|$

Exercise #12 Translate each phrase into a mathematical statement:

a) The sum of – 5 and 12 and 6   _____

b) 14 added to the sum of – 19 and – 4 .   _____

c) The difference between 4 and – 8   _____

d) The sum of 9 and – 4, decreased by 7.   _____

e) 12 less than the difference between 8 and – 5.   _____

f) The product of – 9 and 2, added to 9.   _____

g) Twice the product of – 1 and 6, subtracted from – 4.

_____

h) The quotient of – 12 and the sum of – 5 and – 1 . _____

Sums, Terms, Products, and Factors
Prime and Composite Numbers

Sum is the word we use for _____.

The numbers to be added in the sum are called _____.

Product is the word we use for _____.

The numbers being multiplied are called _____.

Definition   **a is divisible by b** ($a \vdots b$) or **b divides a** ($b \mid a$) if, when dividing a by b,

_____.

Equivalent statements a is divisible by b _____

a is a multiple of b _____

b divides a _____

b is a factor of a _____

b is a divisor of a _____

Exercise #13 List all the factors of:

20:    _____

5:    _____

12:    _____

17:    _____

Property   The number _____ is a factor of any number.   _____

Any nonzero number is a factor of _____.   _____

Definition   A prime number is a natural number (excluding 1) that is divisible only by _____

A natural number greater than 1 that is not prime is called _____

The Set of Prime Numbers: _____

Tests for divisibility: A number is divisible by

          2 if its last digit is divisible by 2.

          3 if the _____is divisible by 3.

          4 if the number formed by its _____ is divisible by 4.

          5 if the _____ is _____.

          8 if the last three digits form a number divisible by 8.

          9 if the sum of its digits is divisible by 9.

          10 if its _____ is _____.

Exercise #14

   a) List all the factors of 24:

   b) List all the prime factors of 24:

   c) List some multiples of 2:

   d) List all the factors of 2:

   e) Find the prime factorization of each number: 15, 28, 108, 1200"

(Section 1.3)

Algebraic Expressions

<u>Definition</u>   A **variable** is a symbol (usually a letter) that stands for a number (or numbers).

Variables can be **used**:

         (1) <u>in equations</u>   -   variables represents unknown quantities
                                 -   the variable is holding the place of a particular number (or
                                      numbers) that has not yet been identified but which needs to be
                                      found.

                                    _____

         (2) <u>in general statements</u> - the variable describes a general relationship between
                                        numbers and/or arithmetic operations.

                                    _____

<u>Definition</u>   A **constant** is a symbol whose value is fixed.

<u>Definition</u>   An algebraic expression is a finite number of additions, subtractions, multiplications,

           and divisions of _____.

Note: An algebraic expression DOES NOT contain the _____ sign.

<u>Definition</u>   An **equation** is a statement that two algebraic expressions are equal.

<u>Definition</u>   The process of replacing the variable(s) in an algebraic expression with
             specific values and evaluating the result is called **algebraic substitution.**

<u>Exercise #15</u> Evaluate the following expressions if $x = 2, y = -3, z = -1$:

    a) $\dfrac{|xy|}{3z}$

    b) $\dfrac{3y^2 - x^2 + 1}{y|z|}$

    c) $yz^3 - (xy)^3$

Exercise #16

Translate each of the following algebraically:

1) Eight more than three times a number.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

_____

2) Three times the sum of eight and a number.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

3) Two less than five times a number is 18.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

4) The sum of two numbers is four le ss than their product.

Choose a variable to represent
the unknown(s) _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

5) Two less than five times a number is 18.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

6) Four more than a number.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

Equivalent phrase: _____
_____

7) Four less than a number is 12.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐
_____

8) The product of a number and seven more than the number.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

9) The product of two more than a number and six less than the number.

Choose a variable to represent
the unknown _____

Translate: _____

Identify the statement:
Algebraic Expression ☐ or Equation ☐

Exercise #16 Write the following:

a) 8 as a power with base 2.

b) 16 as a power with base (-2).

c) -81 as a power with exponent 4.

d) 10,000 as a power with exponent 2

e) -125 as a power with base (-5)

Exercise #17 Find the following:

a) $(-1)^{327} + (-1)^{504} - (-1)^{1003}$

b) $(-1)^n + 3(-1)^{n-1} - 4 \cdot (-1)^{n+2}$; discussion after n.

Exercise #18 Find the following:

a) Find the set $A = \{x | x \in Z, -3 \le x < 2\}$.

b) Find x such that $\dfrac{3}{x} \in N$, where $x \in Z$.

c) Find x such that $\dfrac{-3}{x} \in N$, where $x \in Z$.

More exercises 1.2 (Exponents)

Exercise #19 Simplify:

a) $x^2 x^6$

b) $a^3 a^4$

c) $(x^2 y) xy^2$

d) $\dfrac{x^{14}}{x^6}$

e) $(x^2 x^5)(xx^2 x^{10})$

f) $(3x^2)(5x^4)$

g) $(-2y)(5y^2)(3y^5)$

More exercises 1.7 (The Distributive Property of multiplication over addition)

Exercise #20 Simplify:

a) $\dfrac{1}{2}(6x+8)$

b) $-4(4x+5)-5$

c) $3(4x)$

d) $3(4+x)$

e) $-2(2a-b)$

f) $-(x+ y- z)$

g) $4t - 2(t+2)$

h) $2x-(3y-z)$

i) $10(2a)$

j) $10\left(\dfrac{2}{a}\right)$

k) $10(2+a)$

Exercise # 21 Factor out the greatest common factor (GCF):

a) $-5a+5b$

b) $4a-8b-8c$

c) $10x+10y+20z-40t$

d) $xy-xz$

e) $a^2 b + b^2 c - abc^2$

Exercise #22 Factor out a (-1):

   a) $-a-b$
   b) $-x+y-2z$
   c) $x-5$

More exercises 1.8 (Combining like terms)

Exercise #23 Simplify:

   a) $12b+b$

   b) $12b(b)$

   c) $\dfrac{12b}{b}$

   d) $\dfrac{12+b}{b}$

   e) $4x+5y-9x+y$

   f) $3(x+y)+2x-y$

   g) $x(2+y)-3(xy)+2x(3+y)$

   h) $0.06x+0.09(x+1)$

   i) $8-3[x-4(x-3)]$

   j) $x\{2x^2+x[x-3(x-1)]\}$

   k) $5(x-3y)-x(-3y)$

   l) $-11c-(4-2c)$

   m) $2x-\dfrac{3}{2}y+x-\dfrac{1}{2}y$

   n) $4x(3x^2-y)+(x^3-4xy)+2x(3x)(-4y)$

   o) $-3(x+2)+7(x+2)$

   p) $3-2[2x-5(x+3)]$

   r) $aa^2-3a(a^2+2)$

   s) $-a^2(3a-7)+2a[a^2-4(a-2)]$

   t) $4u^2v(u-v)-(uv^3+u^2v^2)$

   u) $5x(-3y)-x(-3y)$

   v) $5x(x^2+3)+2x(3x^2)$

   x) $100[0.05(x+3)]$

   y) $100+0.05(x+3)$

   z) $100(0.05+x+3)$