REVIEW

Chapter 1 – The Real Number System

In class work: Complete all statements. Solve all exercises.

(Section 1.4)

Definition **A set** is a collection of objects (elements).

The Set of Natural Numbers $\mathbb{N}$

$\mathbb{N}$ = **{ 1, 2, 3, 4, 5,…}**

The Set of Whole Numbers ***W*** $\mathbb{N} \subset W$

W = **{ 0, 1, 2, 3, 4, 5,…}**

The Set of Integers $\mathbb{Z}$ $\mathbb{N} \subset W \subset \mathbb{Z}$

$\mathbb{Z}$ = **{ …, -4, -3, -2, -1 ,0, 1, 2, 3, 4, 5,…}**

The Set of Rational Numbers $\mathbb{Q}$ $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q}$

$$\mathbb{Q} = \left\{ \frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0 \right\}$$

The Set of Irrational Numbers

Examples: $\sqrt{2}, -\sqrt{5}, p$

The Set of Real Numbers $\mathbb{R}$ $\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

$$\mathbb{R} = \{ x \mid x \text{ isrational or } x \text{ is irrational} \}$$

Exercise #1 Decide whether each statement is true or false:

a) Every natural number is positive. true

b) Every whole number is positive. false (0 is neither + nor -)

c) Every integer is a rational number. true

Exercise #2 List all numbers from the set $\left\{ -9, -\sqrt{7},\ -1\frac{1}{4},\ -\frac{3}{5},\ 0,\ \sqrt{5},\ 3,\ 5.9,\ 7 \right\}$ that are

a) natural numbers 3, 7

b) whole numbers but not natural numbers 0

c) odd integers -9, 3, 7

d) rational numbers -9, -1 ¼, -3/5, 0, 3, 5.9, 7

e) irrational numbers $-\sqrt{7}, \sqrt{5}$

(Section 1.2)

Mathematical Symbols

SYMBOL	MEANING	EXAMPLES
$=$	is equal to	
$\neq$	is not equal to	
$\in$	belongs to (about an element)	
$\notin$	it doesn't belong to	
$<$	is less than	
$\leq$	is less than or equal to	
$>$	is greater than	
$\geq$	is greater than or equal to	

Definition   A number ***a*** **is less than a number *b*** $(a < b)$ if a is to the left of b on the number line.

Exercise #3   Write equivalent statements:

a) $2 \leq 3$ $3 \geq 2$

b) $30 > 5$ $5 < 30$

c) $5 > -1 \geq -6$ $-6 \leq -1 < 5$

d) $-4 < -2$ $-2 > -4$

Exercise #4   Fill in the appropriate ordering symbol: either <, >, or =.

a) 2 > -5

b) 19 > 24 – 10

c) 4 – 4 = $4 \cdot 0$

Exercise #5   Write each word statement in symbols:

a) Fifteen is equal to five plus ten. $15 = 5 + 10$

b) Nine is greater than five minus four. $9 > 5 - 4$

c) Sixteen is not equal to nineteen. $16 \neq 19$

d) Two is less than or equal to three. $2 \leq 3$

(Section 1.7)

Properties of Real Numbers

PROPERTIES	ADDITION +	MULTIPLICATION •
COMMUTATIVITY	$a+b=b+a, \quad \forall a,b\in\mathbb{R}$	$ab=ba \quad \forall a,b\in\mathbb{R}$
ASSOCIATIVITY	$(a+b)+c=a+(b+c), \forall a,b,c\in\mathbb{R}$	$(ab)c=a(bc), \quad \forall a,b,c\in\mathbb{R}$
IDENTITY ELEMENT	Zero 0 $a+0=0+a, \forall a\in\mathbb{R}$	One 1 $a\cdot 1=1\cdot a, \forall a\in\mathbb{R}$
INVERSE ELEMENT	$\forall a\in\mathbb{R}$, there is $-a\in\mathbb{R}$ such that $a+(-a)=(-a)+a=0$	$\forall a\in\mathbb{R}, a\neq 0$, there is $\frac{1}{a}\in\mathbb{R}$ such that $a\cdot\frac{1}{a}=\frac{1}{a}\cdot a=1$

<u>Exercise #6</u> Find the opposite and the reciprocal (if any) of each number:

The Number	Its Opposite	Its Reciprocal
2	-2	$\frac{1}{2}$
-4	4	$-\frac{1}{4}$
0	0	none
$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{3}{2}$

The Double Negative Rule

$$-(-a)=a$$

<u>Exercise #7</u> Identify the property used in each example:

a. $(-23)+(-11)=(-11)+(-23)$ commutative property for addition

b. $[123(-2)](-3)=123[(-2)(-3)]$ associative property for multiplication

c. $1\cdot 23=23\cdot 1$ identity element for multiplication

d. $[(-29)+17]+54=(-29)+[(17+54)]$ associative property for addition

e. $(-101)(29)=29(-101)$ commutative property for multiplication

f. $100+0=0+100=100$ identity element for addition

(Section 1.4)

The Absolute Value of a Number

Definition (1) **The absolute value of a number** is the distance between the number and 0 (the origin) on the number line.

$$|a| = dist(a,0)$$

Property $|a| \geq 0, \quad \forall a \in R$

Definition (2)

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Exercise #8 Simplify the following:

a) $|-7| = \mathbf{7}$

b) $-(-7) = \mathbf{7}$

c) $-|-7| = \mathbf{-7}$

d) $-|-(-7)| = \mathbf{-7}$

Exercise #9 Fill in the appropriate ordering symbol: either <, >, or =.

a) $|-3| < |-4|$

b) $3 < |-4|$

c) $-|-6| < -|-4|$

d) $-6 < -(-3)$

e) $-|8| < |-9|$

f) $|6-5| < |2-6|$

(Sections 1.2, 1.5, 1.6)

Operations with Real Numbers

Adding Real Numbers

Same sign
- Add the absolute values of the numbers.
- The sum has the same sign as the given numbers.

Different signs
- Find the difference between the larger absolute value and the smaller.
- The sum has the sign of the number with the larger value.

Subtracting Real Numbers $a - b = a + (-b)$

Multiplying Real Numbers

Dividing Real Numbers $\frac{a}{b} = a \cdot \frac{1}{b}$

Exponent $a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}, \forall n \in \mathbb{N}$

a is called base

n is called power (exponent)

Exercise #10 Simplify the following:

a) $5^2 = 25$

b) $(-5)^2 = 25$

c) $-5^2 = -25$

d) $-(-5)^2 = -25$

e) $2^3 = 8$

f) $(-2)^3 = -8$

g) $-2^3 = -8$

h) $-(-2)^3 = 8$

i) $\left(\frac{1}{3}\right)^2 = \frac{1}{27}$

j) $\left(-\frac{3}{4}\right)^3 = -\frac{27}{64}$

k) $-\left(-\frac{1}{2}\right)^4 = -\frac{1}{16}$

Order of Operations If grouping symbols are present, simplify within them, innermost first, in the following order:

Step 1 powers

Step 2 multiplications and divisions in order from left to right

Step 3 additions and subtractions in order from left to right

Exercise #11 Simplify the following: (See solutions at the end of the handout)

a) $\left|7\cdot 2-8^2\right|$

b) $(-5)^2-3^2+\left|10-2\cdot 3\right|$

c) $-18\div(-3)^2+\left|-8\right|-\left|-4\right|$

d) $\dfrac{(-4)^2-\left|1-2^3\right|}{-(-2)^3+(-1)^{125}}$

e) $\dfrac{\left|-8-4\right|\div\left(2-2^2\right)}{-18\div(-3)^2+\left|-8\right|-\left|-4\right|}$

f) $238\cdot 0-230\div 10+999\div 9-31\cdot 100$

g) $-2(-5)^2+10\div(2)-(-3)^2(2)+4^2\div(-2)$

h) $(4-7)(20-21)^3-2[-10(-3)+2(-1-3)]$

i) $-2(-1)(-7)(-6)+(-2)(-1-7)-3(2-5)$

j) $\left|2-5\right|+\left|7+10\right|-\left|9-12\right|+\left|0-9\right|$

Exercise #12 Translate each phrase into a mathematical statement:

a) The sum of – 5 and 12 and 6 (-5) + 12 + 6

b) 14 added to the sum of – 19 and – 4 . [(-19) + (–4)] + 14

c) The difference between 4 and – 8 4 – (– 8)

d) The sum of 9 and – 4, decreased by 7. [9 + (–4)] – 7

e) 12 less than the difference between 8 and – 5. [8 – (– 5)] – 12

f) The product of – 9 and 2, added to 9. 9 + [(– 9)2]

g) Twice the product of – 1 and 6, subtracted from – 4. (– 4) – 2[(– 1)6]

h) The quotient of – 12 and the sum of – 5 and – 1 . $\dfrac{-12}{(-5)+(-1)}$

Sums, Terms, Products, and Factors

Prime and Composite Numbers

Sum is the word we use for addition.

The numbers to be added in the sum are called terms.

Product is the word we use for multiplication.

The numbers being multiplied are called factors.

Definition: ***a* is divisible by *b*** ($a \vdots b$) or ***b* divides *a*** ($b \mid a$) if, when dividing a by b, the remainder is 0.

Equivalent statements

a is divisible by *b*	6 is divisible by 2
a is a multiple of *b*	6 is a multiple of 2
b divides *a*	2 divides 6
b is a factor of *a*	2 is a factor of 6
b is a divisor of *a*	2 is a divisor of 6

Exercise #13 List all the factors of:

20: 1, 2, 4, 5, 10, 20

5: 1, 5

12: 1, 2, 3, 4, 6, 12

17: 1, 17

Property: The number 1 is a factor of any number. $1 \mid a, \forall a \in \mathbb{R}$

Any nonzero number is a factor of itself. $a \mid a, \forall a \neq 0$

Definition: A prime number is a natural number (excluding1) that is divisible only by 1 and itself.

A natural number greater than 1 that is not prime is called composite.

The Set of Prime Numbers: $\{2,3,5,7,11,13,17,19,23,29,31,...\}$

Tests for divisibility: A number is divisible by

2 if its last digit is divisible by 2.

3 if the sum of its digits is divisible by 3.

4 if the number formed by its last two digits is divisible by 4.

5 if the last digit is 0 or 5.

8 if the last three digits form a number divisible by 8.

9 if the sum of its digits is divisible by 9.

10 if its last digit is 0.

Exercise #14

a) List all the factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

b) List all the prime factors of 24: 2, 3

c) List some multiples of 2: 2, 4, 6, 8, 10, 12, …

d) List all the factors of 2: 1, 2

e) Find the prime factorization of each number: 15, 28, 108, 1200”

3	15
5	5
	1

2	28
2	14
7	7
	1

2	108
2	54
3	27
3	9
3	3
	1

2 * 5	1200
2 * 5	120
2	12
2 * 3	6
	1

$15 = 3 \cdot 5$ $28 = 2^2 \cdot 7$ $108 = 2^2 \cdot 3^3$ $1200 = 2^4 \cdot 3 \cdot 5^2$

(Section 1.3)

Algebraic Expressions

Definition A **variable** is a symbol (usually a letter) that stands for a number (or numbers).

Variables can be **used**:

(1) in equations - variables represents unknown quantities
- the variable is holding the place of a particular number (or numbers) that has not yet been identified but which needs to be found.

$$2x+3=5$$

(2) in general statements - the variable describes a general relationship between numbers and/or arithmetic operations.

$$a+b=b+a$$

Definition A **constant** is a symbol whose value is fixed.

Definition An **algebraic expression** is a finite number of additions, subtractions, multiplications, and divisions of variables and constants.

Note: An algebraic expression DOES NOT contain the = sign.

Definition An **equation** is a statement that two algebraic expressions are equal.

Definition The process of replacing the variable in an algebraic expression with specific values and evaluating the result is called **algebraic substitution.**

Exercise #15 Evaluate the following expressions if $x=2, y=-3, z=-1$:
(See solutions at the end of the handout)

a) $\dfrac{|xy|}{3z}$

b) $\dfrac{3y^2-x^2+1}{y|z|}$

c) $yz^3-(xy)^3$

Exercise #16

Translate each of the following algebraically:

1) Eight more than three times a number.

Choose a variable to represent
the unknown: let x be the number

Translate: $3x + 8$

Identify the statement: Algebraic Expression

---

2) Three times the sum of eight and a number.

Choose a variable to represent
the unknown let a be the number

Translate: $3(8 + a)$

Identify the statement: Algebraic Expression

---

3) Two less than five times a number is 18.

Choose a variable to represent
the unknown let n be the number

Translate: $5n – 2 = 18$

Identify the statement: Equation

---

4) The sum of two numbers is four less than their product.

Choose a variable to represent
the unknown(s): let x be one number,
let y be the other number

Translate: $x + y = xy – 4$

Identify the statement: Equation

---

6) Four more than a number.

Choose a variable to represent
the unknown: let x be the number

Translate: $x + 4$

Identify the statement: Algebraic Expression

---

7) Four less than a number is 12.

Choose a variable to represent
the unknown: let t be the number

Translate: $t – 4 =12$

Identify the statement: Equation

---

8) The product of a number and seven more than the number.

Choose a variable to represent
the unknown: let n be the number

Translate: $n (n + 7)$

Identify the statement: Algebraic Expression

---

9) The product of two more than a number and six less than the number.

Choose a variable to represent
the unknown: let x be the number

Translate: $(x + 2)(x – 6)$

Identify the statement: Algebraic Expression

EXERCISE 11

(a) $|7\cdot 2 - 8^2| = |14-64|$
$= |-50|$
$= 50$

(b) $(-5)^2 - 3^2 + |10-2\cdot 3| =$
$= 25 - 9 + |10-6|$
$= 16 + |4|$
$= 16 + 4$
$= 20$

(c) $-18 \div (-3)^2 + |-8| - |-4| =$
$= -18 \div 9 + 8 - 4$
$= -2 + 8 - 4$
$= 2$

(d) $\dfrac{(-4)^2 - |1-2^3|}{-(-2)^3 + (-1)^{125}} = \dfrac{16 - |1-8|}{-(-8) + (-1)}$

$= \dfrac{16-|-7|}{8-1}$

$= \dfrac{16-7}{7}$

$= \dfrac{9}{7}$

(e) $\dfrac{|-8-4| \div (2-2^2)}{-18 \div (-3)^2 + |-8| - |-4|} =$

$= \dfrac{|-12| \div (2-4)}{-18 \div 9 + 8 - 4}$

$= \dfrac{12 \div (-2)}{-2+8-4} = \dfrac{-6}{2} = -3$

(f) $238\cdot 0 - 230 \div 10 + 999 \div 9 - 31\cdot 100 =$
$= 0 - 23 + 111 - 3100$
$= -23 + 111 - 3100$
$= 88 - 3100$
$= -3012$

(g) $-2(-5)^2 + 10 \div (2) - (-3)^2(2) + 4^2 \div (-2)$
$= -2(25) + 5 - 9\cdot 2 + 16 \div (-2)$
$= -50 + 5 - 18 + (-8)$
$= -45 - 18 - 8$
$= -(45+18+8) = -71$

(h)
$(4-7)(20-21)^3 - 2[-10(-3) + 2(-1-3)] =$
$= (-3)(-1)^3 - 2[30 + 2(-4)]$
$= (-3)(-1) - 2(30-8)$
$= 3 - 2(22)$
$= 3 - 44$
$= -41$

(i)

$-2(-1)(-7)(-6) + (-2)(-1-7) - 3(2-5) =$

$= 2 \cdot 42 + (-2)(-8) - 3(-3)$

$= 84 + 16 + 9$

$= 109$

(e) $yz^3 - (xy)^3 =$

$= (-3)(-1)^3 - (2(-3))^3$

$= (-3)(-1) - (-6)^3$

$= 3 - (-216)$

$= 3 + 216 \quad = 219$

(j) $|2-5| + |7+10| - |9-12| + |0-9| =$

$= |-3| + |17| - |-3| + |-9|$

$= 3 + 17 - 3 + 9$

$= 20 + 6 \quad = 26$

EXERCISE 15

$x = 2, \; y = -3, \quad z = -1$

(a) $\dfrac{|xy|}{3z} = \dfrac{|2(-3)|}{3(-1)}$

$= \dfrac{|-6|}{-3} = \dfrac{6}{-3} = -2$

(b) $\dfrac{3y^2 - x^2 + 1}{y|z|} =$

$= \dfrac{3(-3)^2 - 2^2 + 1}{(-3)\,|-1|}$

$= \dfrac{3 \cdot 9 - 4 + 1}{(-3) \cdot 1}$

$= \dfrac{27 - 4 + 1}{-3} = \dfrac{24}{-3} = -8$