### 6.5 Work and Fluid Forces

Work Done by a Constant Force
When a body moves a distance $d$ along a straight line as a result of being acted on by a force of constant magnitude $F$ in the direction of motion, the work done by the force on the body is defined as:

$$
W=F d
$$

## Example 1: Find the work done if a 20 -pound weight is lifted 6 ft off the ground.

Work Done by a Variable Force Along a Line
The work done by a variable force $F(x)$ in the direction of motion along the $x$-axis from $x=a$ to $x=b$ is

$$
W=\int_{a}^{b} F(x) d x
$$

Example 2: When a particle is located at a distance $x \mathrm{ft}$ from the origin, a force of $x^{2}+2 x$ pounds acts on it. How much work is done in moving it from $x=1$ to $x=3$ ?

Hooke's Law for Springs
The force required to hold a stretched or compressed spring $x$ units from its natural (unstressed) length is proportional to $x$.

$$
F=k x, \text { where } k \text { is the spring constant ( force constant) }
$$

1. A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm . How much work is done in stretching the spring from 15 cm to 18 cm ?
(A: 1.56 J )
2. (Exercise \#2/6.5) A spring has a natural length of 10 in . An $800-\mathrm{lb}$ force stretched the spring to 14 in .
a. Find the force constant.
b. How much work is done in stretching the spring from 10 in to 12 in?
c. How far beyond its natural length will a $1600-\mathrm{lb}$ force stretch the spring? (A: 2001b/in; $33.3 \mathrm{lb}-\mathrm{ft} ; 8 \mathrm{in}$ )
3. (Exercise \#7/6.5) A mountain climber is about to haul up a 50 m length of hanging rope. How much work will it take if the rope weighs $0.624 \mathrm{~N} / \mathrm{m}$ ?
(A: 780 J )
4. (Exercise \#8/6.5) A bag of sand originally weighing 144 lb was lifted at a constant rate. As it rose, sand also leaked out at a constant rate. The sand was half gone by the time the bag had been lifted to 18 ft . How much work was done lifting the sand this far? ( Neglect the weight of the bag and lifting equipment.)
(A: $1944 \mathrm{lb}-\mathrm{ft}$ )

## Pumping Liquids from Containers

How much work does it take to pump all or part of the liquid from a container? To find out how much work is required to pump the liquid, we imagine lifting the liquid out one thin horizontal slab at a time and applying the equation $W=F d$ to each slab. We then evaluate the integral this leads to as the slabs become thinner and more numerous. The integral we get each time depends on the weight of the liquid and the dimensions of the container, but the way we find the integral is always the same.
5. (Exercise \#13/6.5) The rectangular tank shown here, with its top at ground level, is used to catch runoff water. Assume that the water weighs $62.4 \mathrm{lb} / \mathrm{ft}^{3}$.

Ground $10 \mathrm{ft} \quad 12 \mathrm{ft}$
a) How much work does it take to empty the tank by pumping the waterlevel back to ground level once the tank is full?
b) If the water is pumped to ground level with a ( $5 / 11$ )-horsepower (hp) motor (work output $250 \mathrm{ft}-\mathrm{lb} / \mathrm{sec}$ ), how long will it take to empty the full tank?
c) Show that the pump in part (b) will lower the water level 10 ft (halfway) during the first 25 min of pumping.

6. (Exercise \#22/6.5) You are in charge of the evacuation and repair of the storage tank shown here. The tank is a hemisphere of radius 10 ft and is full of benzene weighing $56 \mathrm{lb} / \mathrm{ft}^{3}$. A firm you contacted says it can empty the tank for $1 / 2$ cent per foot-pound of work. Find the work required to empty the tank by pumping the benzene to an outlet 2 ft above the top of the tank. If you have $\$ 5000$ budgeted for the job, can you afford to hire the firm?


## The Pressure-Depth Equation

In a fluid that is standing still, the pressure $p$ at a depth $h$ is the force per unit area exerted by the liquid at that depth. $p=w h$
where $w$ is the fluid's weight-density ( weight per unit volume).
Example At a depth of 10 ft in water, for which the weight-density is $62.4 \mathrm{lb} / \mathrm{ft}^{3}$, the pressure is $62.4 \cdot 10=624 \mathrm{lb} / \mathrm{ft}^{2}$. Hence if a thin, flat plate of area $5 \mathrm{ft}^{2}$ is suspended in a horizontal position at a depth of 10 ft in water, then the water exerts a downward force of $624 \cdot 5=3120 \mathrm{lb}$ on the top face of the plate and an equal upward force on its bottom face.

## Fluid Force on a Constant-Depth Surface

$$
F=p A=w h A
$$

where $F$ is the total force exerted by the fluid against a flat horizontal surface, $p$ is the pressure at the depth $h, A$ is the area of the surface.

Example 3: What is the fluid force at the bottom of a 10 ft by 20 ft rectangular swimming pool of 3 ft deep? The weight-density of freshwater is $62.4 \mathrm{lb} / \mathrm{ft}^{3}$.

It is an important fact that at a given depth in a liquid, the pressure is the same in all directions. However, if a flat plate is submerged in a vertical position in the liquid, then the pressure on the face of the plate is not constant because the pressure increases with increasing depth. Consequently, the total force exerted on a vertical plate must be computed by integration.

What is the force exerted by a fluid against one side of a vertical plate submerged in a fluid of weight-density $w$ ?

## The Integral for Fluid Force Against a Vertical Flat Plate

Suppose that a plate submerged vertically in fluid of weight-density $w$ runs from $y=a$ to $y=b$ on the $y$-axis. Let $L(y)$ be the length of the horizontal strip measured from left to right along the surface of the plate at level $y$. Then the force exerted by the fluid against one side of the plate is

$$
F=\int_{a}^{b} w \cdot(\text { strip depth }) \cdot L(y) d y
$$

7. (Exercise \#35/6.5) In a pool filled with water to a depth of 10 ft , calculate the fluid force on one side of a 3 ft by 4 ft rectangular plate if the plate rests vertically at the bottom of the pool on its 4 ft edge.
8. (Exercise \#38/6.5) The isosceles triangle is submerged vertically so that part of the plate sticks out of the lake, as shown here. What force does the water exert on one face of the plate now?
(A: 114.4 lb )


More practice:
9. (Exercise \#6/ 6.5) A bathroom scale is compressed $1 / 16$ in when a $150-\mathrm{lb}$ person stands on it. Assuming that the scale behaves like a spring that obeys Hooke's Law, how much does someone who compresses the scale $1 / 8$ in weigh? How much work is done compressing the scale $1 / 8$ in?
(A: $25 / 16 \mathrm{ft} \cdot \mathrm{lb}$ )
10. A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m . It is filled with water to a height of 8 m . Find the work required to empty the tank by pumping all the water to the top of the tank. (The density of water is $1000 \mathrm{~kg} / \mathrm{m}^{3}$.)
(A: $3.4 \times 10^{6} \mathrm{~J}$ )
11. A damn has the shape of a trapezoid. The height is 20 m and the width is 50 m at the top and 30 m at the bottom. Find the force on the dam due to hydrostatic pressure if the water level is 4 m from the top of the dam. (The density of water is $1000 \mathrm{~kg} / \mathrm{m}^{3}$.)

$$
\text { (A: } 4.43 \times 10^{7} N \text { ) }
$$

12. A cable that weighs $2 \mathrm{lb} / \mathrm{ft}$ is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done.
(A: 650,000 ft-lb)
13. (Exercise \#20/6.5) A right-circular cylindrical tank of height 10 ft and radius 5 ft is lying horizontally and is full of diesel fuel weighing $53 l b s / f t^{3}$. How much work is required to pump all the fuel to a point 15 ft above the top of the tank?
