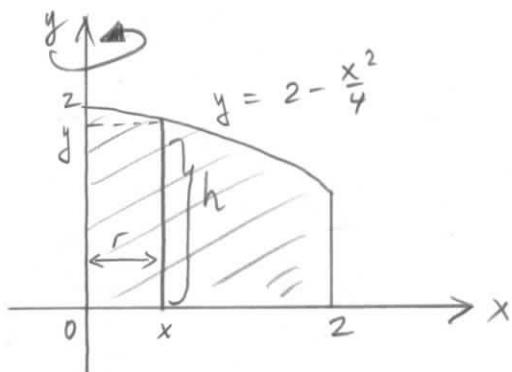


## 6.2 HANDOUT

①  
②



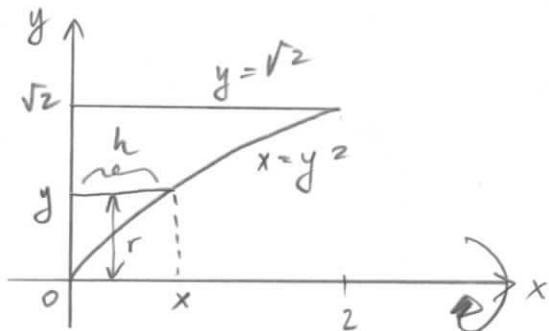
We'll use vertical shells, // y-axis  
from  $x=0$  to  $x=2$

$$V = \int_0^2 2\pi (\text{shell radius}) (\text{shell height}) dx$$

shell radius =  $r = x$   
shell height =  $h = y = 2 - \frac{x^2}{4}$

$$\begin{aligned} V &= \int_0^2 2\pi x (2 - \frac{x^2}{4}) dx \\ &= 2\pi \int_0^2 (2x - \frac{x^3}{4}) dx \\ &= 2\pi \left( x^2 \Big|_0^2 - \frac{1}{4} \cdot \frac{x^4}{4} \Big|_0^2 \right) \\ &= 2\pi (4 - \frac{1}{16} \cdot 16) = 6\pi \\ V &= 6\pi \text{ cubic units} \end{aligned}$$

③

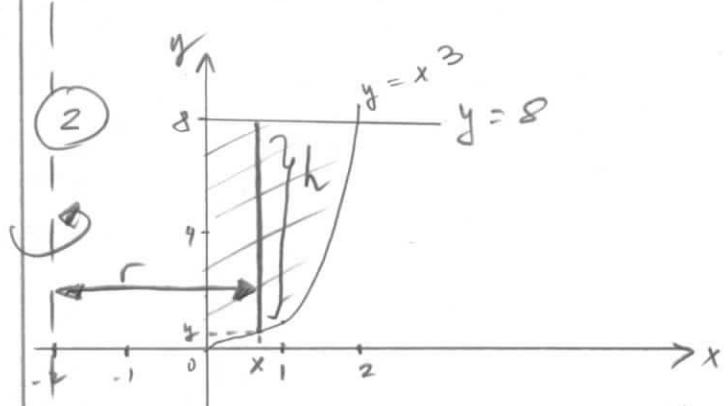


We'll use horizontal shells, // x-axis  
from  $y=0$  to  $y=\sqrt{2}$

$$V = \int_0^{\sqrt{2}} 2\pi (\text{shell radius}) (\text{shell height}) dy$$

shell radius =  $r = y$   
shell height =  $h = x = y^2$

$$\begin{aligned} V &= \int_0^{\sqrt{2}} 2\pi y \cdot y^2 dy = 2\pi \int_0^{\sqrt{2}} y^3 dy \\ &= 2\pi \left[ \frac{y^4}{4} \right]_0^{\sqrt{2}} = \frac{\pi}{2} \cdot 4 = 2\pi \\ V &= 2\pi \text{ cubic units.} \end{aligned}$$



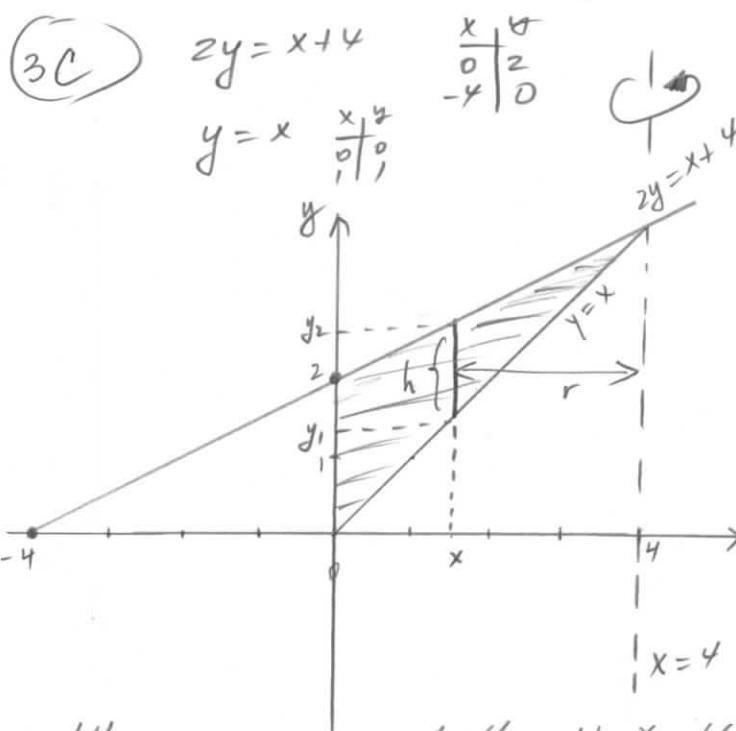
$x=-2$  we'll use vertical shells,  
parallel to  $x=-2$

from  $x=0$  to  $x=2$

$$V = \int_0^2 2\pi (\text{shell radius}) (\text{shell height}) dx$$

shell radius =  $r = 2+x$   
shell height =  $h = 8-y = 8-x^3$

$$\begin{aligned} V &= 2\pi \int_0^2 (2+x)(8-x^3) dx \\ &= 2\pi \int_0^2 (16-2x^3+8x-x^4) dx \\ &= 2\pi \left( \left[ 6x - 2\frac{x^4}{4} + 8\frac{x^2}{2} - \frac{x^5}{5} \right]_0^2 \right) \\ &= 2\pi (32 - \frac{1}{2} \cdot 16 + 4 \cdot 4 - \frac{1}{5} \cdot 32) \\ &= 2\pi (40 - \frac{32}{5}) \\ &= 2\pi \cdot \frac{168}{5} = \frac{336}{5}\pi \\ V &= \frac{336}{5}\pi \text{ cubic units.} \end{aligned}$$



We'll use vertical shells, ||  $x=4$   
from  $x=0$  to  $x=4$

$$V = \int_0^4 2\pi \left( \frac{\text{shell}}{r} \right) \left( \frac{\text{shell}}{h} \right) dx$$

$$\text{shell radius } r = 4 - x$$

$$\text{shell height } h = y_2 - y_1$$

where  $y_2$  from  $2y = x + 4$   
 $y_1$  from  $y = x$

$$\therefore h = \left( \frac{x}{2} + 2 \right) - x = 2 - \frac{x}{2}$$

$$V = \int_0^4 2\pi (4-x) \left( 2 - \frac{x}{2} \right) dx$$

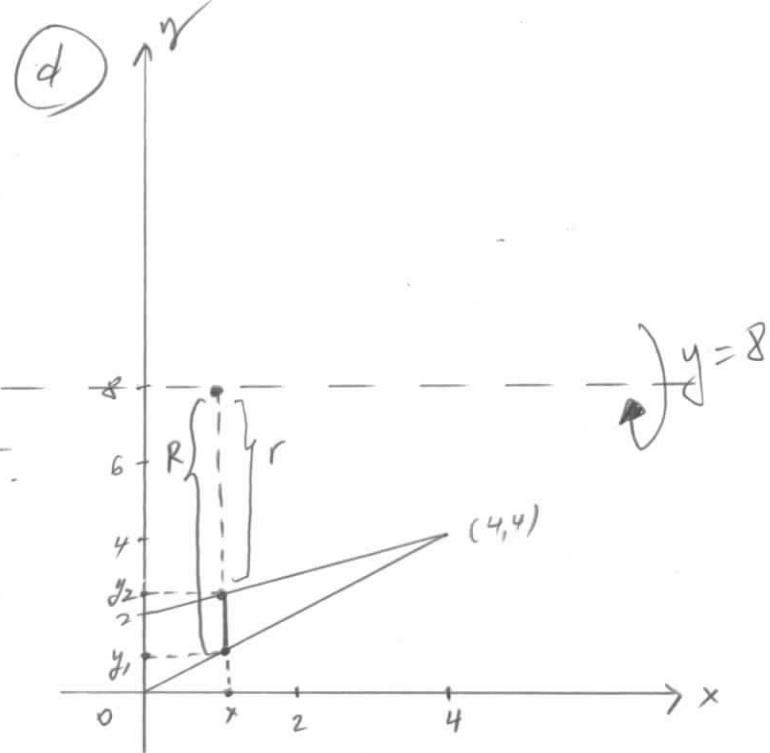
$$= 2\pi \int_0^4 \left( 8 - 4x + \frac{x^2}{2} \right) dx$$

$$= 2\pi \left( [8x]_0^4 - 4 \cdot \frac{x^2}{2} \Big|_0^4 + \frac{1}{2} \cdot \frac{x^3}{3} \Big|_0^4 \right)$$

$$= 2\pi \left( 32 - 2 \cdot 16 + \frac{1}{6} \cdot 64 \right)$$

$$= \frac{64\pi}{3}$$

$$V = \frac{64\pi}{3} \text{ cubic units.}$$



We'll use vertical slabs, ⊥  $y=8$   
from  $x=0$  to  $x=4$

$$V = \int_0^4 A(x) dx$$

where  $A(x) = \text{area of the cross-section (washer)}$

$$A(x) = \pi (R^2 - r^2)$$

$$R = 8 - y_1 = 8 - x$$

$$r = 8 - y_2 = 8 - \left( \frac{x}{2} + 2 \right) = 6 - \frac{x}{2}$$

$$A(x) = \pi \left( (8-x)^2 - (6-\frac{x}{2})^2 \right)$$

$$= \pi \left( 28 - 10x + \frac{3}{4}x^2 \right)$$

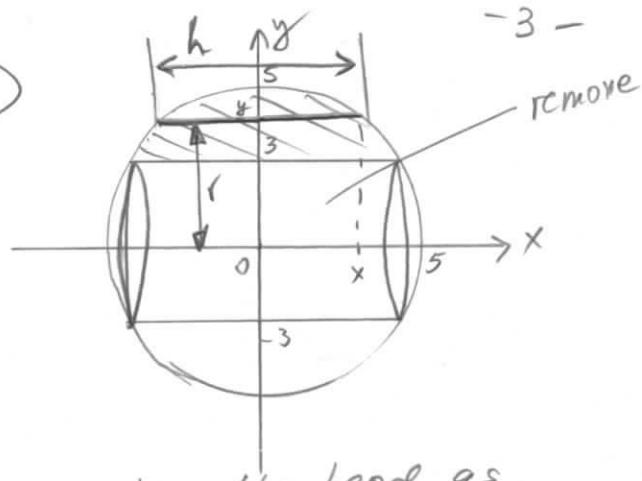
$$\text{Then, } V = \int_0^4 \pi \left( 28 - 10x + \frac{3}{4}x^2 \right) dx$$

$$= \pi \left( \left[ 28x - 10 \cdot \frac{x^2}{2} + \frac{3}{4} \cdot \frac{x^3}{3} \right]_0^4 \right)$$

$$= \pi \left( 28 \cdot 4 - 5 \cdot 16 + \frac{1}{4} \cdot 64 \right)$$

$$V = 48\pi \text{ cubic units.}$$

(4)



(a)

we can view the bead as being formed by revolving the shaded region about  $x$ -axis.

We'll use horizontal shells // to  $x$ -axis, from  $y=3$  to  $y=5$

$$V = \int_3^5 2\pi (\text{shell})(\text{shell}) dy$$

$$\text{shell radius} = r = y$$

$$\text{shell height} = h = 2x \rightarrow \text{where}$$

$$x^2 + y^2 = 25$$

$$x = \sqrt{25 - y^2}$$

$$\therefore h = 2\sqrt{25 - y^2}$$

$$V = \int_3^5 2\pi y (2\sqrt{25 - y^2}) dy$$

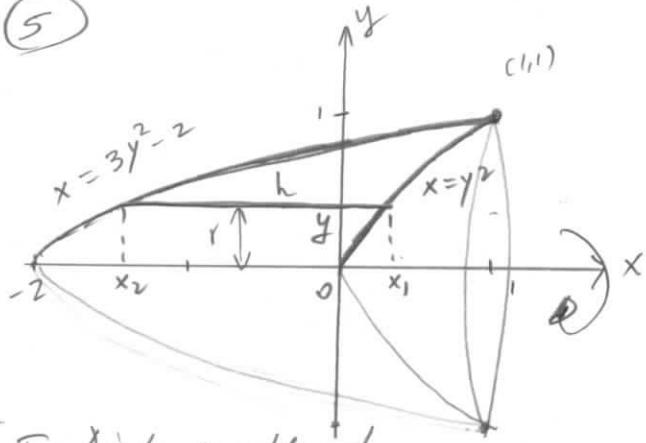
$$= 2\pi \int_3^5 2y \sqrt{25 - y^2} dy$$

$$\text{let } \begin{cases} 25 - y^2 = u \\ -2y dy = du \\ y=3 \Rightarrow u=16 \\ y=5, \quad u=0 \end{cases}$$

$$= 2\pi \int_0^{16} -\sqrt{u} du = 2\pi \int_0^{16} \sqrt{u} du$$

$$= 2\pi \left[ \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{16} = \frac{4\pi}{3} \cdot 64 = \frac{256}{3}\pi$$

(5)



I Disk method

We'll use vertical slabs, ⊥  $x$ -axis

$$V = V_1 - V_2, \begin{cases} V_1 = \int_{-2}^1 A_1(x) dx, \\ A_1(x) = \pi y^2 = \pi \left( \frac{x+2}{3} \right)^2 \end{cases}$$

$$\begin{cases} V_2 = \int_0^1 A_2(x) dx \\ A_2(x) = \pi y^2 = \pi x^2 \end{cases}$$

II Washer method

We'll use vertical slabs, ⊥  $x$ -axis

$$V = V_1 + V_2, \begin{cases} V_1 = \int_{-2}^0 A_1(x) dx, \\ A_1(x) = \pi y^2 = \pi \left( \frac{x+2}{3} \right)^2 \end{cases}$$

$$\begin{cases} V_2 = \int_0^1 A_2(x) dx \\ A_2(x) = \pi \left( \frac{x+2}{3} - x \right)^2 \end{cases}$$

III Shell method

We'll use horizontal shells, //  $x$ -axis

from  $y=0$  to  $y=1$

$$V = \int_0^1 2\pi (\text{shell})(\text{shell}) dy$$

$$\begin{aligned} r &= y, \quad h = x_1 - x_2 \\ &= y^2 - (3y^2 - 2) \end{aligned}$$

$$h = 2 - 2y^2$$

$$V = 2\pi \int_0^1 y (2 - 2y^2) dy = \frac{1}{3}\pi$$

$V = \frac{1}{3}\pi$  cubic units