6.1 Volumes Using Cross-Sections

In-class work:

1. Show that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

2. Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere. (A: $\frac{5\pi}{12} r^3$)

3. (Exercise #1 / 6.1) Find the volume of the solid that lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross-sections perpendicular to the axis on the interval $0 \le x \le 4$ are squares whose diagonals un from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$. (A: 16)

4. (Exercise #11 / 6.1) Find the volume of a tetrahedron with three mutually perpendicular faces and three mutually perpendicular edges with lengths 3, 4, and 5 cm. (A: 10)

5. (Exercise #16 / 6.1) Find the volume of the solid generated by revolving the region bounded by $x = \frac{3y}{2}$, the y-axis, and $y = 2$ about the y-axis. (A: 6π)

6. (Exercise #20 / 6.1) Find the volume of the solid generated by revolving the region bounded by the lines and curves given about the x-axis.

   $y = x^3, y = 0, x = 2$ (A: $\frac{128\pi}{7}$)

7. (Exercise #29 / 6.1) Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec x \tan x$, and on the left by the y-axis, about the line $y = \sqrt{2}$. (A: $\frac{\pi^2}{2} - \frac{11\pi}{3} + 2\sqrt{2\pi}$)

8. (Exercise #42 / 6.1) Find the volume of the solid generated by revolving the region bounded by $y = 4 - x^2$ and $y = 2 - x$ about the x-axis. (A: $\frac{108\pi}{5}$)

9. (Exercise #46 / 6.1) Find the volume of the region enclosed by the triangle with vertices (0,1), (1,0), and (1,1) about the y-axis. (A: $\frac{2\pi}{3}$)