12.2 Vectors

Definitions

1. **A vector** is a directed line segment.

2. Two vectors are **equal** if they have the same length and direction.

3. A vector is in **standard position** if its initial point is at the origin.

4. If $\vec{v}$ is a two-dimensional vector in the plane equal to the vector with initial point at the origin and terminal point (v_1, v_2), then the **component form** of $\vec{v}$ is

$$\vec{v} = < v_1, v_2 >$$

5. If $\vec{v}$ is a three-dimensional vector equal to the vector with initial point at the origin and terminal point (v_1, v_2, v_3), then the **component form** of $\vec{v}$ is

$$\vec{v} = < v_1, v_2, v_3 >$$

Notes

1. Given the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the standard position vector equal to $\overrightarrow{PQ}$ is

$$\vec{v} = < x_2 - x_1, y_2 - y_1, z_2 - z_1 >$$

2. Two vectors are equal if and only if their standard position vectors are identical.

Vector Algebra

Let $\vec{u} = < u_1, u_2, u_3 >$ and $\vec{v} = < v_1, v_2, v_3 >$ be vectors with *k* a scalar.

Definition

Addition: $\vec{u} + \vec{v} = < u_1 + v_1, u_2 + v_2, u_3 + v_3 >$

Scalar multiplication: $k\vec{u} = < ku_1, ku_2, ku_3 >$

Properties

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors and *a*, *b* be scalars.

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
3. $\vec{u} + \vec{0} = \vec{u}$
4. $\vec{u} + (-\vec{u}) = \vec{0}$
5. $0\vec{u} = \vec{0}$
6. $1\vec{u} = \vec{u}$
7. $a(b\vec{u}) = (ab)\vec{u}$
8. $a(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v}$
9. $(a + b)\vec{u} = a\vec{u} + b\vec{u}$

Exercise 1 Find the vector represented by the directed line segment with initial point $A(2, -3, 4)$ and terminal point $B(-2, 1, 1)$. Find its magnitude.

Exercise 2 If $\vec{u} = < 4, 0, 3 >$ and $\vec{v} = < -2, 1, 5 >$, find $|\vec{u}|, \vec{u} + \vec{v}, \vec{u} - \vec{v}, 3\vec{v}, 2\vec{u} + 5\vec{v}$.

Unit Vectors

Definition A vector of length 1 is called a unit vector.

Standard unit vectors

Exercise 3 $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 4\vec{i} + 7\vec{k}$. Express $2\vec{a} + 3\vec{b}$ in terms of the standard unit vectors.

Exercise 4 Find the unit vector in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$.

Exercise 5 Express the vector $2\vec{i} + \vec{j} - 2\vec{k}$ as a product of its length and direction.

Exercise 6 Find the component form of the unit vector in a plane that makes an angle $q = \frac{2p}{3}$ with the positive x-axis.

Exercise 7 A 100-lb weight hangs from two wires that form angles of 50° and 32°, respectively, with the horizontal. Find the tension forces in both wires and their magnitudes.

Exercise 8 Find a vector with representation given by the directed line segment $\overrightarrow{AB}$, where $A(2,3), B(-2,1)$.

Exercise 9 Find a vector that has the same direction as $< -2,4,2 >$, but length 6.

Exercise 10 Find the direction of $\overrightarrow{P_1P_2}$, if $P_1(-1,1,5), P_2(2,5,0)$.

Exercise 11 If a child pulls a sled through the snow with a force of 50 N exerted at an angle of 38° above the horizontal, find the horizontal and vertical components of the force.

Exercise 12 Two forces with magnitudes 10 lb and 12 lb act o an object at P. The first force makes an angle of 45° with the horizontal, while the second force makes an angle of 30°. Find the resulting force acting at P. Find its magnitude and its direction * by finding the angle made with the horizontal).