12.1 Three-Dimensional Coordinate System

In-class work:

1. What surfaces in R^3 are represented by the equations:
   a. $z = 3$ b. $y = 5$

2. Give a geometric description of the set of points in R^3 that satisfy the equations:
   a. $x = -1, z = 0$ b. $x + y = 2$ c. $y = x$

3. Plot the points: $(0,5,2), (4,0,-1), (2,4,6), (1,-1,2)$.

4. Which of the points $P(6,2,3), Q(-5,-1,4)$ and $R(0,3,8)$ is closest to the *xz*-plane? Which one lies in the *yz*-plane?

The Distance Formula

If $P_1(x_1, y_1, z_1), P_2(x_2, y_2, z_2)$, then the distance between the two points is

$$|P_1P_2| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

The Equation of a Sphere with Center $C(x_0, y_0, z_0)$ and radius r is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

5. Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is a sphere and find its center and radius.

6. Determine if the points lie on a straight line: $A(5,1,3), B(7,9,-1)$ and $C(1,-15,11)$.

7. Find the distance from $(3,7,-5)$ to
   a. *xy*-plane c. *yz*-plane
   b. *xz*-plane d. *x*-axis

8. Find the equation of a sphere with center $(1,-4,3)$ and radius 5. What is the intersection of the sphere with the xz-plane?

9. Describe the region in R^3 given by :
   a. $x > 3$
   b. $0 \le z \le 6$
   c. $x^2 + y^2 + z^2 > 1$

10. Write inequalities to describe the region:
    a. Half-space consisting of all points to the left of *xz*-plane.
    b. The solid rectangular box in the first octant bounded by the planes $x = 1$, $y = 2$, $z = 3$
    c. The region of all points between (but not on) the spheres of radius r and R centered at the origin.

11. Let P such that the distance from *P* to $A(-1,5,3)$ is twice the distance from *P* to $B(6,2,-2)$. Show that the set of all such points is a sphere, and find its radius and center.