11.1 Parametrizations of plane curves

In-class work:

1. Sketch and identify the curve defined by the parametric equations $x = t^2 - 2t$, $y = t + 1$, $t \in R$.

2. What curve is represented by the parametric equations $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$?

3. What curve is represented by the parametric equations $x = \sin t$, $y = \cos t$, $0 \le t \le 2\pi$?

4. What curve is represented by the parametric equations $x = \sin 2t$, $y = \cos 2t$, $0 \le t \le 2\pi$?

5. Sketch the curve with parametric equations $x = \sin t$, $y = \sin^2 t$.

6. (Exercise #14/11.1) Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation. Indicate the portion of the graph traced by the particle and the direction of motion.
$$x = \sqrt{t+1},\ y = \sqrt{t},\ t \ge 0$$

7. (Exercise #11/11.1) Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation. Indicate the portion of the graph traced by the particle and the direction of motion.
$$x = t^2,\ y = t^6 - 2t^4,\ t \in R$$

8. (Exercise #8/11.1) Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation. Indicate the portion of the graph traced by the particle and the direction of motion.
$$x = 4\sin t,\ y = 5\cos t,\ 0 \le t \le 2\pi$$

9. Find a parametrization for the line through (a,b), having slope *m*.

10. (Exercise #22/11.1) Find a parametrization for the line segment with endpoints $(-1,3)$ and $(3,-2)$.

11. (Exercise #19/11.1) Find parametric equations and a parameter interval for the motion of a particle that starts at $(a,0)$ and traces the circle $x^2 + y^2 = a^2$

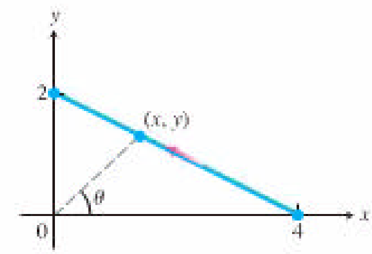
   a. once clockwise
   b. twice clockwise.

12. (Exercise #28/11.1) Find parametric equations and a parameter interval for the motion of a particle that moves along the graph of $y = x^2$ in the following way: beginning at $(0,0)$ it moves to $(3,9)$, and then travels back and forth from $(3,9)$ to $(-3,9)$ infinitely many times.

13. The Cycloid   A wheel of radius a rolls along a horizontal straight line. Find parametric equations for the path traced by a point P on the wheel's circumference.

- One of the first people to study the cycloid was Galileo, who proposed that bridges be built in the shape of cycloids and who tried to find the area under one arch of a cycloid.
- Later this curve arose in connection with the *brachistochrone problem*: Find the curve along which a particle will slide in the shortest time (under the influence of gravity) from a point A to a lower point B not directly beneath A. The Swiss mathematician John Bernoulli, who posed this problem in 1696, showed that among all possible curves that join A to B, the particle will take the least time sliding from A to B if the curve is part of an inverted arch of a cycloid.
- The Dutch physicist Huygens had already shown that the cycloid is also the solution to the *tautochrone problem*; that is, no matter where a particle P is placed on an inverted cycloid, it takes the same time to slide to the bottom. Huygens proposed that pendulum clocks (which he invented) should swing in cycloidal arcs because then the pendulum would take the same time to make a complete oscillation whether it swings through a wide or a small arc.

14. (Exercise #31/11.1) Find a parametrization for the line segment joining points $(0,2)$ and $(4,0)$ using the angle q in the accompanying figure as the parameter.



15. a) Sketch the curve represented by the parametric equations $x = e^t$, $y = \sqrt{t}$, $0 \le t \le 1$, and indicate with a an arrow the direction in which the curve is traced as t increases.
b) Eliminate the parameter to find a Cartesian equation of the curve.

16. Find parametric equations for the path of a particle that moves counterclockwise halfway around the circle $(x-2)^2 + y^2 = 4$, from the top to the bottom.