

# ASSIGNMENT

## SECTION 5.5

$$(7) \int (1 - \cos \frac{t}{2})^2 \sin \frac{t}{2} dt = I$$

$$\text{let } 1 - \cos \frac{t}{2} = u$$

$$\frac{1}{2} \cdot \sin \frac{t}{2} dt = du$$

$$\Rightarrow \sin \frac{t}{2} dt = 2 du$$

$$I = \int 2u^2 du = 2 \frac{u^3}{3} + C$$

$$\boxed{I = \frac{2}{3} (1 - \cos \frac{t}{2})^3 + C}$$

$$(16) \int \frac{3 dx}{(2-x)^2}$$

$$\text{let } 2-x = u$$

$$-dx = du \Rightarrow dx = -du$$

$$\int \frac{3 dx}{(2-x)^2} = - \int \frac{3 du}{u^2}$$

$$= -3 \int u^{-2} du$$

$$= -3 \frac{u^{-2+1}}{-2+1} + C$$

$$= 3(2-x)^{-1} + C$$

$$\boxed{= \frac{3}{2-x} + C}$$

$$(22) \int \tan^2 x \sec^2 x dx = I$$

$$\text{let } \tan x = u$$

$$\sec^2 x dx = du$$

$$I = \int u^2 du = \frac{u^3}{3} + C$$

$$\boxed{I = \frac{1}{3} \tan^3 x + C}$$

$$(34) \int \frac{\cos \sqrt{\theta}}{\sqrt{\theta} \sin^2 \sqrt{\theta}} d\theta = I$$

$$\text{let } \sin \sqrt{\theta} = u$$

$$(\cos \sqrt{\theta}) \cdot \frac{1}{2\sqrt{\theta}} d\theta = du$$

$$\Rightarrow \frac{\cos \sqrt{\theta}}{\sqrt{\theta}} d\theta = 2 du$$

$$I = \int \frac{2 du}{u^2} = 2 \int u^{-2} du$$

$$= 2(-u^{-2+1}) + C$$

$$= -2(\sin \sqrt{\theta})^{-1} + C$$

$$\boxed{I = \frac{-2}{\sin \sqrt{\theta}} + C}$$

$$(40) \int (\sin 2\theta) e^{\sin^2 \theta} d\theta = I$$

$$\text{let } \sin^2 \theta = u$$

$$2 \sin \theta \cos \theta d\theta = du$$

$$\sin 2\theta d\theta = du$$

$$I = \int e^u du = e^u + C$$

$$\boxed{I = e^{\sin^2 \theta} + C}$$

$$(52) \int \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx = I$$

$$\text{let } \tan^{-1} x = u$$

$$\frac{1}{1+x^2} dx = du$$

$$I = \int \sqrt{u} du = \int u^{\frac{1}{2}} du$$

$$I = \frac{u^{\frac{1}{2}+1}}{\frac{3}{2}} + C$$

$$I = \frac{2}{3} (\tan^{-1} x)^{\frac{3}{2}} + C$$

$$\boxed{I = \frac{2}{3} \sqrt{(\tan^{-1} x)^3} + C}$$

### SECTION 5.6

$$(28) I = \int_0^{\pi/3} \frac{4 \sin \theta}{1-4 \cos \theta} d\theta$$

$$\text{let } 1-4 \cos \theta = u$$

$$4 \sin \theta d\theta = du$$

$$\text{when } \theta = 0, u = 1-4 = -3$$

$$\theta = \frac{\pi}{3}, u = 1-4 \cdot \frac{1}{2} = -1$$

$$I = \int_{-3}^{-1} \frac{du}{u} = \ln |u| \Big|_{-3}^{-1}$$

$$I = \ln |-1| - \ln |-3| = \ln 1 - \ln 3$$

$$\boxed{I = -\ln 3 = \ln \frac{1}{3}}$$

$$(40) \int_1^{e^{\pi/4}} \frac{4 dt}{t(1+\ln^2 t)} = I$$

$$\text{let } \ln t = u$$

$$\frac{1}{t} dt = du$$

$$\text{when } t=1, u=0$$

$$t = e^{\pi/4}, u = \ln e^{\pi/4} = \frac{\pi}{4}$$

$$I = 4 \int_0^{\pi/4} \frac{du}{1+u^2} = 4 \tan^{-1} u \Big|_0^{\pi/4}$$

$$I = 4 (\tan^{-1} \frac{\pi}{4} - \tan^{-1} 0)$$

$$I = 4 (\tan^{-1} \frac{\pi}{4} - 0)$$

$$\boxed{I = 4 \tan^{-1} \frac{\pi}{4}}$$

$$(16) \int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dy}{y\sqrt{9y^2-1}} = 1$$

$$\text{let } 3y = u \Rightarrow y = \frac{u}{3}$$

$$3dy = du \Rightarrow dy = \frac{1}{3} du$$

$$\text{when } y = -\frac{2}{3}, u = 3\left(-\frac{2}{3}\right) = -2$$

$$y = -\frac{\sqrt{2}}{3}, u = 3\left(-\frac{\sqrt{2}}{3}\right) = -\sqrt{2}$$

$$1 = \int_{-2}^{-\sqrt{2}} \frac{1}{3} \cdot \frac{du}{\frac{u}{3}\sqrt{u^2-1}}$$

$$1 = \int_{-2}^{-\sqrt{2}} \frac{3}{3} \frac{du}{u\sqrt{u^2-1}}$$

Recall that

$$(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}} \quad \text{if } x > 1$$

$$(\sec^{-1} x)' = \frac{1}{-x\sqrt{x^2-1}} \quad \text{if } x < -1$$

Therefore,

$$1 = -\left. \sec^{-1} x \right]_{-2}^{-\sqrt{2}}$$

$$1 = -(\sec^{-1}(-\sqrt{2}) - \sec^{-1}(-2))$$

$$\sec^{-1}(-\sqrt{2}) = x \in [0, \pi] \quad \text{iff}$$

$$\sec x = -\sqrt{2} \quad \text{iff}$$

$$\frac{1}{\cos x} = -\sqrt{2} \quad \text{iff}$$

$$\cos x = -\frac{\sqrt{2}}{2} \quad \text{iff}$$

$$x = \frac{3\pi}{4}$$

$$\text{so } \sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$$

$$\text{similarly, } \sec^{-1}(-2) = \frac{2\pi}{3}$$

Therefore,

$$1 = -\left(\frac{3\pi}{4} - \frac{2\pi}{3}\right)$$

$$\boxed{1 = -\frac{\pi}{12}}$$