MATH 180 - 12th Ed. 5.5 - SELECTED PROBLEMS

(4) let $x^4+1=u$; $I=\frac{-1}{x^4+1}+C$

(10) let $1-\cos\frac{t}{2}=u$;
$I=\frac{2}{3}(1-\cos\frac{t}{2})^3+C$

(16) $I=\frac{2}{5}\sqrt{5x+8}+C$

(22) let $3z+4=u$;
$I=\frac{1}{3}\sin(3z+4)+C$

(28) let $7-\frac{r^5}{10}=u$; $I=\frac{-1}{2}(7-\frac{r^5}{10})^4+C$

(34) let $\sqrt{t}+3=u$; $I=2\sin(\sqrt{t}+3)+C$

(40) let $1-\frac{1}{x^2}=u$; $I=\frac{1}{3}(1-\frac{1}{x^2})^{3/2}+C$

(46) let $x-5=u$;
$I=\frac{3}{7}(x-5)^{7/3}+\frac{15}{2}(x-5)^{4/3}+C$

(52) let $\sin^2\theta=u$; $I=e^{\sin^2\theta}+C$

(58) let $x^2=u$; $I=\frac{1}{2}\sec^{-1}(x^2)+C$

(60) let $e^\theta=u$; $I=\sec^{-1}(e^\theta)+C$

(64) let $\tan^{-1}x=u$; $I=\frac{2}{3}\sqrt{(\tan^{-1}x)^3}+C$

(70) let $\cos\sqrt{\theta}=u$; $I=\frac{4}{\sqrt{\cos\sqrt{\theta}}}+C$

(76) let $\tan 2x=u$
Then $\frac{dy}{dx}=\int 4\sec^2 2x\tan 2x\,dx$
$=\int u(2du)=\tan^2 2x+C_1$

Find $C_1=4$
$\frac{dy}{dx}=\tan^2 2x+4=\sec^2 2x+3$
$y=\int(\sec^2 2x+3)\,dx=$
$=\frac{1}{2}\tan 2x+3x+C_2$
find $C_2=-1$
$y=\frac{1}{2}\tan 2x+3x-1$

SECTION 5.6

SELECTED ANSWERS

(4) $I=2$

(10) a) $I=\frac{\sqrt{10}-3}{2}$; b) $I=\frac{3-\sqrt{10}}{2}$

(16) let $1+\sqrt{y}=u$; $I=\int_2^3 u^{-2}du=\frac{1}{6}$

(22) let $y^3+6y^2-12y+9=u$
$I=\int_9^4\frac{1}{3}u^{-\frac{1}{2}}du=-\frac{2}{3}$

(28) let $1-4\cos\theta=u$
$I=\int_{-3}^{-1}\frac{1}{u}du=\ln\frac{1}{3}$

(34) let $\sin t=u$;
$I=\int_{\frac{1}{\sqrt{2}}}^{1}\frac{1}{u}du=\ln\sqrt{2}$

(40) let $\ln t=u$;
$I=4\int_0^{\pi/4}\frac{du}{1+u^2}=4\tan^{-1}\frac{\pi}{4}$

(46) let $u = 3y$

$$I = \int_{-2}^{-\sqrt{2}} \frac{du}{u\sqrt{u^2-1}} = -\int_{-2}^{-\sqrt{2}} \frac{du}{|u|\sqrt{u^2-1}}$$

$$= -\sec^{-1}u \Big]_{-2}^{-\sqrt{2}} = -\left(\sec^{-1}(-\sqrt{2}) - \sec^{-1}(-2)\right)$$

$$= -\frac{\pi}{12}$$

(50) let $\pi + \pi \sin x = u$

$$I = 2\int_0^{\pi} \frac{\pi}{2}(\sin u)\frac{1}{\pi}\,du = 2$$

(52) $A = \frac{4\pi}{3}$

(54) $A = \frac{1}{12}$

(58) $A = \int_0^1 x^2\,dx + \frac{1}{2}(1)(1) = \frac{5}{6}$

(64) $A = \int_{-1}^{3} (2x - x^2 + 3)\,dx = \frac{32}{3}$

(76) $A = 3\int_{-1}^{1} (1-y^2)\,dy = 4$

(82) $A = \int_{-1}^{2} (x^3 - 3x^2 + 4)\,dx = \frac{27}{4}$

(88) $A = A_1 + A_2 = 2A_1$

$$A = 2\int_0^1 \left(\sin\frac{\pi x}{2} - x\right)dx = \frac{4-\pi}{\pi}$$

(94) $A = A_1 + A_2 = 2A_2$

$$A = 2\int_0^1 (x^3 - x^5)\,dx = \frac{1}{6}$$

(96) $A = \int_0^{\pi/4} (\cos x - \sin x)\,dx$

$$= \sqrt{2} - 1$$

(100) $\int_0^{2\ln 2} \left(e^{x/2} - e^{-x/2}\right)dx = 1$

(106) $A = A_1 + A_2$

$$A_1 = 2\int_0^1 y^{\frac{1}{2}}\,dy = \frac{4}{3}$$

$$A_2 = \int_1^2 [3 - y - (y-1)^2]\,dy = \frac{7}{6}$$

so $A = \frac{5}{2}$

SECTION 7.1

(4) $I = \ln|4r^2 - 5| + C$

(10) $\frac{(\ln x)^4}{8}\Big]_1^4 = \frac{(\ln 4)^4}{8}$

(18) let $\ln^2 x + 1 = u$

$$I = \int \frac{1}{2\sqrt{u}}\,du = \sqrt{\ln^2 x + 1} + C$$

(34) let $\ln x = u$;

$$I = \int_0^{\ln 2} 2^u\,du = \frac{2^{\ln 2} - 1}{\ln 2}$$

(38) let $\ln x = u$

$$I = \int_0^{\ln 4} \frac{1}{\ln 2}\,u\,du = \ln 4$$

SECTION 8.1

(5) $I = -x^2\cos x + 2x\sin x + 2\cos x + C$

(10) $I = \left(\frac{1}{2}x^2 - \frac{3}{2}x + \frac{5}{4}\right)e^{2x} + C$

(12) $\left[\begin{array}{ll} \text{let } f = \sin^{-1}y, & g' = 1 \\ \text{then } f' = \frac{1}{\sqrt{1-y^2}} & g = y \end{array}\right.$

$I = y\sin^{-1}y - \int \frac{y\,dy}{\sqrt{1-y^2}} =$

$= y\sin^{-1}y + \sqrt{1-y^2} + C$

(14) $I = y\tan y - \ln|\sec y| + C$

where $y = 2x$

$I = 2x\tan 2x - \ln|\sec 2x| + C$

(16) $I = (-p^4 - 4p^3 - 12p^2 - 24p - 24)e^{-p} + C$

(22) $I = \frac{1}{2}\left(e^{-y}\sin y - e^{-y}\cos y\right) + C$

(24) $I = -\frac{e^{-2x}}{4}(\sin 2x + \cos 2x) + C$

(26) $I = \frac{2}{3}\left[-\frac{2}{5}(1-x)^{\frac{5}{2}}\right]_0^1 = \frac{4}{15}$

(28) $I = x\ln(x+x^2) - 2x +$
$+ \ln|x+1| + C$

(30) $I = \frac{z^2}{4}\left[2(\ln z)^2 - 2\ln z + 1\right] + C$

(34) $I = \frac{-1}{\ln x} + C$

(40) $I = \frac{-1}{3}\cos x^3 + C$

(46) $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$

I let $\sqrt{x} = u$ or

II let $e^{\sqrt{x}} = u \Rightarrow \sqrt{x} = \ln u$

then $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}\,dx = du$

$e^{\sqrt{x}}\,dx = 2\sqrt{x}\,du$

$e^{\sqrt{x}}\,dx = 2\ln u\,du$

$I = \int (\ln u)(2\ln u\,du) =$

$= 2\int (\ln u)^2\,du$

integration by parts:

$\left[\begin{array}{ll} \text{let } f = (\ln u)^2 & g' = 1 \\ \text{then } f' = \frac{2\ln u}{u} & g = u \end{array}\right.$

$I = 2\left[u(\ln u)^2 - 2\int \ln u\,du\right]$

int. by parts

$\left[\begin{array}{ll} \text{let } f = \ln u & g' = 1 \\ \text{then } f' = \frac{1}{u} & g = u \end{array}\right.$

$I = 2u(\ln u)^2 - 4\left(u\ln u - \int du\right)$

$I = 2u(\ln u)^2 - 4u\ln u + 4u + C$

$I = 2e^{\sqrt{x}}(\sqrt{x})^2 - 4\sqrt{x}\,e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$

$I = 2e^{\sqrt{x}}x - 4\sqrt{x}\,e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$