Math 180

SECTION 4.6

(22) $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\ln(\csc x)}{\left(x-\frac{\pi}{2}\right)^2} = \dfrac{0}{0}$ (l'Hopital)

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\left(\ln(\csc x)\right)'}{\left(\left(x-\frac{\pi}{2}\right)^2\right)'}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\frac{-\cancel{\csc x}\cot x}{\cancel{\csc x}}}{2\left(x-\frac{\pi}{2}\right)}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{-\cot x}{2\left(x-\frac{\pi}{2}\right)} = \dfrac{0}{0}$ (l'Hopital)

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\csc^2 x}{2} = \dfrac{1^2}{2} = \boxed{\dfrac{1}{2}}$

(28) $\lim\limits_{\theta\to 0} \dfrac{\left(\frac{1}{2}\right)^{\theta}-1}{\theta} = \dfrac{0}{0}$ (l'H)

$= \lim\limits_{\theta\to 0} \dfrac{\left(\frac{1}{2}\right)^{\theta}\ln\frac{1}{2}}{1} = \left(\frac{1}{2}\right)^{0}\ln\frac{1}{2}$

$= \ln\frac{1}{2} = \ln 1 - \ln 2 = \boxed{-\ln 2}$

(31) $\lim\limits_{x\to\infty} \dfrac{\ln(x+1)}{\log_2 x} = \dfrac{\infty}{\infty}$ (l'Hopital)

$= \lim\limits_{x\to\infty} \dfrac{\left(\ln(x+1)\right)'}{\left(\log_2 x\right)'}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{1}{x+1}}{\frac{1}{x\ln 2}}$

$= \ln 2 \lim\limits_{\infty} \dfrac{x}{x+1}$ (l'Hopitol)

$= \ln 2 \lim\limits_{\infty} \dfrac{1}{1} = \boxed{\ln 2}$

(34) $\lim\limits_{x\to 0^+} \dfrac{\ln(e^x-1)}{\ln x} = \dfrac{\infty}{\infty}$ (l'H)

$= \lim\limits_{x\to 0^+} \dfrac{\left(\ln(e^x-1)\right)'}{\left(\ln x\right)'}$

$= \lim\limits_{0^+} \dfrac{\frac{1}{e^x-1}\cdot e^x}{\frac{1}{x}}$

$= \lim\limits_{0^+} \dfrac{xe^x}{e^x-1} = \dfrac{0}{0}$ (l'Hopital)

$= \lim\limits_{0^+} \dfrac{(xe^x)'}{(e^x-1)'}$

$= \lim\limits_{0^+} \dfrac{e^x+xe^x}{e^x} = \dfrac{1+0}{1} = \boxed{1}$

(40) $\lim_{x\to 0^+}\left(\frac{3x+1}{x} - \frac{1}{\sin x}\right) = \infty - \infty$

$= \lim_{0^+} \frac{(3x+1)\sin x - x}{x\sin x} = \frac{0}{0}$ (l'H)

$= \lim_{0^+} \frac{((3x+1)\sin x - x)'}{(x\sin x)'}$

$= \lim_{0^+} \frac{3\sin x + (3x+1)\cos x - 1}{\sin x + x\cos x} = \frac{0}{0}$

$= \lim_{0^+} \frac{3\cos x + 3\cos x - (3x+1)\sin x}{\cos x + \cos x - x\sin x}$

$= \frac{3\cdot\cos 0 + 3\cos 0 - (3\cdot 0+1)\sin 0}{2\cos 0 - 0\sin 0}$

$= \frac{3+3-0}{2-0} = \frac{6}{2} = \boxed{3}$

(56) $\lim_{x\to 0^+}\left(1+\frac{1}{x}\right)^x = \infty^0$

let $f(x) = \left(1+\frac{1}{x}\right)^x = e^{\ln(1+\frac{1}{x})^x}$

Then,

$\lim_{0^+} f(x) = \lim_{0^+} e^{\ln(1+\frac{1}{x})^x}$

$= e^{\lim_{0^+} \ln(1+\frac{1}{x})^x}$

$= e^{\lim_{0^+} x\ln(1+\frac{1}{x})}$

Calculate

$\lim_{0^+} x\ln(1+\frac{1}{x}) = 0\cdot(\infty)$

$= \lim_{0^+} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = \frac{\infty}{\infty}$ (l'Hopital)

$= \lim_{0^+} \frac{(\ln(1+x^{-1}))'}{(x^{-1})'}$

$= \lim_{0^+} \frac{\frac{1}{1+\frac{1}{x}}(-\cancel{x^{-2}})}{(-1)\cancel{x^{-2}}}$

$= \lim_{0^+} \frac{1}{1+\frac{1}{x}} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$

Therefore,

$\lim_{0^+} f(x) = e^0 = \boxed{1}$

(65) f - continuous at $x=0$

iff $\lim_{x\to 0} f(x) = f(0) = c$

$c = \lim_{0} f(x) = \lim_{0} \frac{9x - 3\sin 3x}{5x^3}$ (l'Hopital $\frac{0}{0}$)

$= \lim_{0} \frac{9 - 3\cos 3x \cdot 3}{15x^2}$

$= \lim_{x\to 0} \frac{9 - 9\cos 3x}{15x^2} = \frac{0}{0}$ (l'H)

$= \lim_{x\to 0} \frac{9\sin 3x \cdot 3}{30x} = \frac{0}{0}$ (l'H)

$= \lim_{x\to 0} \frac{27\cos 3x \cdot 3}{30} = \frac{81}{30} = \frac{27}{10}$

$\boxed{c = \frac{27}{10}}$