# SECTION 4.2 – SELECTED PROBLEMS

(1) Find $c \in [0,1]$ such that $f'(c) = \frac{f(1)-f(0)}{1-0}$, where $f(x) = x^2+2x-1$ ...

Verify that the Mean Value Theorem can be applied:

$$\left\{\begin{array}{l} f = \text{cont. on } [0,1] \\ f = \text{diff. on } (0,1) \end{array}\right.$$

$\Rightarrow \exists c \in (0,1)$ such that

$$\left.\begin{array}{l} f'(c) = \frac{f(1)-f(0)}{1-0} \\ \text{but } f'(x) = 2x+2 \end{array}\right\} \Rightarrow$$

$$2c+2 = \frac{2-(-1)}{1} \Rightarrow c = \frac{1}{2}$$

(6) We need to see if $f(x) = x^{4/5}$, $x \in [0,1]$ is continuous on $[0,1]$ and differentiable on $(0,1)$

$f(x) = x^{4/5}$ is continuous for $\forall x$.

$$f'(x) = \frac{4}{5}x^{-\frac{1}{5}} = \frac{4}{5\sqrt[5]{x}}$$

$f'(x)$ exists for $\forall x \neq 0$. So $f$ is diff. on $(0,1)$

so, yes, $f(x) = x^{4/5}$ satisfies the hypotheses of the Mean Value Theorem.

(10) $f(x) = \begin{cases} 3, & x=0 \\ x^2+3x+a, & 0<x<1 \\ mx+b, & 1 \le x \le 2 \end{cases}$

$f$ cont. on $[0,2]$ iff $f$ cont. at $x=0$ and $x=1$

$f$ cont. at $x=0$ iff $\lim\limits_{x\to 0} f(x) = f(0) \Rightarrow a=3$

$f$ cont. at $x=1$ iff $\lim\limits_{x\to 1} f(x) = f(1) \Rightarrow m+b=5$

$f$ diff. on $(0,2)$ iff $f$ diff. at $x=1$

$f'_l(1) = f'_r(1) \Rightarrow m=1$

Therefore, $a=3$, $m=1$, $b=4$

(15) $f(x) = x^4 + 3x + 1$, $x \in [-2,-1]$

We'll use the intermediate Value Theorem.

First, we look for a change in sign.

We note that $f(-2) > 0$ and $f(-1) < 0$, so $\exists\, c \in (-2,-1)$ such that $f(c) = 0$

Second, we'll show that we cannot have more than one $c$ with that property.

We'll assume there are two $c$'s with that property.

$c_1, c_2 \in (-2,-1)$ such that $f(c_1) = f(c_2) = 0$

By Rolle's theorem, $\exists\, c \in (c_1, c_2)$ where $f'(c) = 0$

But this is impossible since $f'(x) = 4x^3 + 3$ and when $-2 \le x \le -1$

$$-8 \le x^3 \le -1$$
$$-32 \le 4x^3 \le -4$$
$$-29 \le 4x^3 + 3 \le -1$$

$f'(x) < 0 \;\forall x \in [-2,-1]$

Therefore, our assumption is false $\Rightarrow$ $f$ has only one zero in $[-2,-1]$.

(35) $f'(x) = e^{2x} = \left(e^{2x} \cdot \frac{1}{2}\right)' \Rightarrow$

$f(x) = \frac{1}{2}e^{2x} + c$, $c \in \mathbb{R}$

$f(0) = \frac{3}{2} \Rightarrow c = 1$

so $f(x) = \frac{1}{2}e^{2x} + 1$