

(180) SECTION 3.1

$$(7) K(z) = \frac{1-z}{2z}$$

$$K'(z) = \lim_{h \rightarrow 0} \frac{K(z+h) - K(z)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1-(z+h)}{2(z+h)} - \frac{1-z}{2z}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1-z-h}{2(z+h)} - \frac{1-z}{2z}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{z(1-z-h) - (1-z)(z+h)}{2h(z)(z+h)}$$

$$= \lim_{h \rightarrow 0} \frac{z - z^2 - zh - (z+h - z^2 - zh)}{2hz(z+h)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{z} - z^2 - zh - \cancel{z} - h + \cancel{z^2} + zh}{2hz(z+h)}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{2hz(z+h)}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{2z(z+h)} = \frac{-1}{2z \cdot 2}$$

$$\boxed{K'(z) = \frac{-1}{2z^2}}$$

$$K'(-1) = \frac{-1}{2(-1)^2} = \frac{-1}{2}$$

$$K'(1) = \frac{-1}{2(1)^2} = \frac{-1}{2}$$

$$K'(\sqrt{2}) = \frac{-1}{2(\sqrt{2})^2} = \frac{-1}{4}$$

$$(10) v = t - \frac{1}{t}$$
$$\frac{dv}{dt} = \lim_{h \rightarrow 0} \frac{v(t+h) - v(t)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{t+h - \frac{1}{t+h} - (t - \frac{1}{t})}{h}$$

$$= \lim_{h \rightarrow 0} \frac{t+h - \frac{1}{t+h} - t + \frac{1}{t}}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{h}{h} + \frac{\frac{1}{t} - \frac{1}{t+h}}{h} \right)$$

$$= 1 + \lim_{h \rightarrow 0} \frac{t+h - t}{h t (t+h)}$$

$$= 1 + \lim_{h \rightarrow 0} \frac{h}{h t (t+h)}$$

$$= 1 + \lim_{h \rightarrow 0} \frac{1}{t(t+h)}$$

$$= 1 + \frac{1}{t^2}$$

$$\boxed{\frac{dv}{dt} = 1 + \frac{1}{t^2}}$$

$$(16) \quad y = (x+1)^3, \quad x = -2 \quad -2-$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h+1)^3 - (x+1)^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{((x+1)+h)^3 - (x+1)^3}{h}$$

$$\left(\text{Note: } (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \right)$$

$$= \lim_{h \rightarrow 0} \frac{(x+1)^3 + 3(x+1)^2h + 3h^2(x+1) + h^3 - (x+1)^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(x+1)^2h + 3(x+1)h + h^2}{h}$$

$$= \lim_{h \rightarrow 0} (3(x+1)^2 + 3(x+1) + h)$$

$$= 3(x+1)^2$$

$$\text{So } \left| y' = \frac{dy}{dx} = 3(x+1)^2 \right|$$

$$m = \left. \frac{dy}{dx} \right|_{x=-2} = 3(-2+1)^2 = \boxed{3}$$

$$(22) \quad w = z + \sqrt{z}$$

$$f(z) = w = z + \sqrt{z}$$

$$w' = \frac{dw}{dz} = \lim_{h \rightarrow 0} \frac{f(z+h) - f(z)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{z+h + \sqrt{z+h} - (z + \sqrt{z})}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h + \sqrt{z+h} - \sqrt{z}}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{h}{h} + \frac{\sqrt{z+h} - \sqrt{z}}{h} \right)$$

$$= 1 + \lim_{h \rightarrow 0} \frac{\sqrt{z+h} - \sqrt{z}}{h} \cdot \frac{\sqrt{z+h} + \sqrt{z}}{\sqrt{z+h} + \sqrt{z}}$$

$$= 1 + \lim_{h \rightarrow 0} \frac{z+h - z}{h(\sqrt{z+h} + \sqrt{z})}$$

$$= 1 + \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{z+h} + \sqrt{z})}$$

$$= 1 + \lim_{h \rightarrow 0} \frac{1}{\sqrt{z+h} + \sqrt{z}} = 1 + \frac{1}{2\sqrt{z}}$$

$$\left| w' = \frac{dw}{dz} = 1 + \frac{1}{2\sqrt{z}} \right|$$

$$\left. \frac{dw}{dz} \right|_{z=4} = 1 + \frac{1}{2\sqrt{4}} = 1 + \frac{1}{4} = \boxed{\frac{5}{4}}$$