SOLUTIONS

SECTION 2.7

(24) $g(x) = x^3 - 3x$

Want $g'(x) = 0$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} =$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - 3(x+h) - x^3 + 3x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{3x}-3h-\cancel{x^3}+\cancel{3x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2-3)}{\cancel{h}}$$

$$= \lim_{h\to 0} (3x^2+3xh+h^2-3)$$

$= 3x^2 - 3$, so $g'(x) = 3x^2 - 3$

let $g'(x) = 0$

$3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

if $x = 1, y = -2$ $\quad$ if $x = -1, y = 2$

$(1,-2)$ $\quad$ $(-1,2)$

The graph of $g(x) = x^3 - 3x$ will have horizontal tangents at $\boxed{(1,-2) \text{ and } (-1,2)}$

(32) $g(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$P(0,0)$

The graph has a tangent at $(0,0)$ if $g'(0)$ exists

$$g'(0) = \lim_{x\to 0} \frac{g(x)-g(0)}{x-0}$$

$$= \lim_{x\to 0} \frac{x\sin\frac{1}{x}}{x}$$

$= \lim_{x\to 0} \sin\frac{1}{x}$ doesn't exist b/c $\sin\frac{1}{x}$ oscillates ~~too~~ much ~~between 1 and -1~~ near 0

So, the graph doesn't have a tangent at $(0,0)$.