HOMEWORK – SOLUTIONS

SECTION 2.3

(8) We want

$|f(x)-L| < \varepsilon$ for any x; $|x-x_0|<\delta$

$|-\frac{3}{2}x+3-7.5| < \varepsilon$ for $|x+3| < \delta$

$-\delta < x+3 < \delta$

$\boxed{-\delta-3 < x < \delta-3}$

From the graph, $-\delta-3 = -3.1$, so $\delta = 0.1$

$\delta-3 = -2.9$, so $\delta = 0.1$

So, $\boxed{\delta = 0.1}$

SECTION 2.4

(4) $f(x) = \begin{cases} 3-x, & x<2 \\ 2, & x=2 \\ \frac{x}{2}, & x>2 \end{cases}$

(a) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x}{2} = \frac{2}{2} = 1$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(3-x) = 3-2 = 1$

$f(2) = 2$

(b) Yes, $\lim_{x\to 2} f(x) = 1$ because

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = 1$

(c) $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+}(3-x) = 3-(-1) = 4$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-}(3-x) = 3-(-1) = 4$

(d) Yes, $\lim_{x\to -1} f(x) = 4$ because

$4 = \lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x)$

(6) (a) Yes, $\lim_{x\to 0^+} g(x) = 0$

by the Squeeze Theorem

since $\underbrace{-\sqrt{x} \le g(x) \le \sqrt{x}}$ when $x>0$

(from the given graph)

Without a graph:

$g(x) = \sqrt{x}\sin\frac{1}{x}$

$-1 \le \sin\frac{1}{x} \le 1$ for any $x \ne 0$

$-\sqrt{x} \le \sqrt{x}\sin\frac{1}{x} \le \sqrt{x}$

$\lim_{x\to 0^+} \sqrt{x} = \lim_{x\to 0^+} -\sqrt{x} = 0$

So, by the Squeeze Theorem,

$\lim_{x\to 0^+} g(x) = 0$

(b) No, $\lim_{x\to 0^-} g(x)$ does not exist

since $\sqrt{x}$ is not defined when $x<0$

(c) No, $\lim_{x\to 0} g(x)$ does not exist

since $\lim_{x\to 0^-} g(x)$ does not exist.

Continue section 2.4

(17) $\lim_{x \to -2^+} (x+3)\dfrac{|x+2|}{x+2}$

$|x+2| = \begin{cases} x+2 & \text{if } x \geq -2 \\ -(x+2) & \text{if } x < -2 \end{cases}$

when $x \to -2^+$, $x > -2$, so

$\lim_{x \to -2^+} (x+3)\dfrac{|x+2|}{x+2} =$

$\lim_{x \to -2^+} (x+3)\dfrac{x+2}{x+2} =$

$\lim_{x \to -2^+} (x+3) = -2+3 = \boxed{1}$

(24) $\lim_{h \to 0^-} \dfrac{h}{\sin 3h} = \lim_{h \to 0^-} \dfrac{3h}{\sin 3h} \cdot \dfrac{1}{3}$

$= \dfrac{1}{3} \lim_{h \to 0^-} \dfrac{1}{\frac{\sin 3h}{3h}} = \dfrac{1}{3} \cdot 1 = \boxed{\dfrac{1}{3}}$

(30) $\lim_{x \to 0} \dfrac{x^2 - x + \sin x}{2x} =$

$= \dfrac{1}{2} \lim_{x \to 0} \dfrac{x^2 - x + \sin x}{x}$

$= \dfrac{1}{2} \lim_{x \to 0} \left(\dfrac{x^2}{x} - \dfrac{x}{x} + \dfrac{\sin x}{x}\right)$

$= \dfrac{1}{2} \lim_{x \to 0} \left(x - 1 + \dfrac{\sin x}{x}\right)$

$= \dfrac{1}{2}(0 - 1 + 1) = \dfrac{1}{2}(0) = \boxed{0}$

(33) $\lim_{\theta \to 0} \dfrac{\sin\theta}{\sin 2\theta} =$

$= \lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} \cdot \dfrac{2\theta}{\sin 2\theta} \cdot \dfrac{\cancel{\theta}}{2\cancel{\theta}}$

$= \lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} \cdot \lim_{\theta \to 0} \dfrac{1}{\frac{\sin 2\theta}{2\theta}} \cdot \lim_{\theta \to 0} \dfrac{1}{2}$

$= 1 \cdot 1 \cdot \dfrac{1}{2} = \boxed{\dfrac{1}{2}}$

(43) $\lim_{x \to \infty} \dfrac{\sin 2x}{x}$

$-1 \leq \sin 2x \leq 1$ for any x

when $x \to \infty$, $\dfrac{1}{x} > 0$

$-\dfrac{1}{x} \leq \dfrac{1}{x}\sin 2x \leq \dfrac{1}{x}$

$\lim_{x \to \infty} \dfrac{1}{x} = \lim_{x \to -\infty} \dfrac{1}{x} = 0$

By the Squeeze Th, $\boxed{\lim_{x \to \infty} \dfrac{\sin 2x}{x} = 0}$

(64) $\lim_{x \to +\infty} \dfrac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}} =$

(Divide numerator and denominator by x^{-2}, the highest power of den.)

$= \lim_{x \to +\infty} \dfrac{\frac{x^{-1}}{x^{-2}} + \frac{x^{-4}}{x^{-2}}}{\frac{x^{-2}}{x^{-2}} - \frac{x^{-3}}{x^{-2}}}$

$= \lim_{x \to +\infty} \dfrac{x + \frac{1}{x^2}}{1 - \frac{1}{x}} = \dfrac{+\infty + 0}{1 - 0} = \boxed{\infty}$

when $x \to -\infty$, $\frac{1}{x} \to 0$