HOMEWORK - SOLUTIONS

SECTION 2.1

(32) $g(t) = 2 + \cos t$

a) Find the average rate of change of g over $[0, \pi]$

$$\frac{\Delta g}{\Delta t} = \frac{g(\pi) - g(0)}{\pi - 0}$$

$$= \frac{(2+\cos\pi) - (2+\cos 0)}{\pi}$$

$$= \frac{(2-1)-(2+1)}{\pi} = \frac{1-3}{\pi}$$

$$\frac{\Delta g}{\Delta t} = \frac{-2}{\pi}$$

b) Find the average rate of change of g over $[-\pi, \pi]$

$$\frac{\Delta g}{\Delta t} = \frac{g(\pi) - g(-\pi)}{\pi - (-\pi)}$$

$$= \frac{(2+\cos\pi) - (2+\cos(-\pi))}{2\pi}$$

$$= \frac{(2-1) - (2-1)}{2\pi} = \frac{1-1}{2\pi} = 0$$

$$\frac{\Delta g}{\Delta t} = 0$$

(36) a) $P(10, 80)$

Q	slope of $PQ = \frac{\Delta y}{\Delta t}$
$Q_1(5, 20)$	$\frac{80-20}{10-5} = 12$ m/sec
$Q_2(7, 39)$	$\frac{80-39}{10-7} = 13.7$ m/sec
$Q_3(8.5, 58)$	$\frac{80-58}{10-8.5} = 14.7$ m/sec
$Q_4(9.5, 72)$	$\frac{80-72}{10-9.5} = 16$ m/sec

b) approx. 16 m/sec.

SECTION 2.2

(28) $\lim_{v\to 2} \frac{v^3-8}{v^4-16} = \lim_{v\to 2} \frac{v^3-2^3}{(v^2)^2-4^2}$

$= \lim_{v\to 2} \frac{(v-2)(v^2+2v+2^2)}{(v^2-4)(v^2+4)}$

$= \lim_{v\to 2} \frac{\cancel{(v-2)}(v^2+2v+4)}{\cancel{(v-2)}(v+2)(v^2+4)}$

$= \lim_{v\to 2} \frac{v^2+2v+4}{(v+2)(v^2+4)} = \frac{2^2+2(2)+4}{(2+2)(2^2+4)}$

$= \frac{12}{32} = \boxed{\frac{3}{8}}$

(34) $\lim_{x\to -2} \frac{x+2}{\sqrt{x^2+5}-3} =$

$= \lim_{x\to -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{(\sqrt{x^2+5}-3)(\sqrt{x^2+5}+3)}$

$= \lim_{x\to -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{(\sqrt{x^2+5})^2-3^2}$

$= \lim_{x\to -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{x^2+5-9}$

$= \lim_{x\to -2} \frac{(x+2)(\sqrt{x^2+5}+3)}{x^2-4}$

$= \lim_{x\to -2} \frac{\cancel{(x+2)}(\sqrt{x^2+5}+3)}{\cancel{(x+2)}(x-2)}$

$= \lim_{x\to -2} \frac{\sqrt{x^2+5}+3}{x-2} = \frac{\sqrt{(-2)^2+5}+3}{-2-2}$

$= \frac{3+3}{-4} = \frac{-6}{4} = \boxed{\frac{-3}{2}}$

(46) $f(x) = \frac{1}{x}$, $x=-2$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$

$\lim_{h\to 0} \frac{f(-2+h)-f(-2)}{h} =$

$\lim_{h\to 0} \frac{\frac{1}{-2+h} - \frac{1}{-2}}{h} =$

$\lim_{h\to 0} \frac{{}^{2/}\frac{1}{h-2} + {}^{h-2/}\frac{1}{2}}{h} =$

$\lim_{h\to 0} \frac{\frac{\cancel{2}+h-\cancel{2}}{2(h-2)}}{h} =$

$\lim_{h\to 0} \frac{\cancel{h}}{2\cancel{h}(h-2)} = \lim_{h\to 0} \frac{1}{2(h-2)} =$

$= \frac{1}{2(0-2)} = \boxed{\frac{-1}{4}}$

(48) $f(x) = \sqrt{3x+1}$, $x=0$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} \frac{\sqrt{3h+1}-\sqrt{1}}{h} = \lim_{h\to 0} \frac{\sqrt{3h+1}-1}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{3h+1}-1)(\sqrt{3h+1}+1)}{h(\sqrt{3h+1}+1)}$

$= \lim_{h\to 0} \frac{(\sqrt{3h+1})^2-1^2}{h(\sqrt{3h+1}+1)}$

$= \lim_{h\to 0} \frac{3h+\cancel{1}-\cancel{1}}{h(\sqrt{3h+1}+1)}$

$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}(\sqrt{3h+1}+1)} = \lim_{h\to 0} \frac{3}{\sqrt{3h+1}+1}$

$= \frac{3}{\sqrt{0+1}+1} = \boxed{\frac{3}{2}}$