Section 4.3 – How derivatives affect the shape of a graph
Section 4.5 – Curve sketching

Definition 1 Let f be a function defined on an interval I.

***f* is increasing** on I if and only if for any $x_1 < x_2$, $f(x_1) < f(x_2)$

***f* is decreasing** on I if and only if for any $x_1 < x_2$, $f(x_1) > f(x_2)$

Increasing/Decreasing Test

Let f be a differentiable function on I.

If $f'(x) > 0$ for any $x \in I$, then f is increasing on I.

If $f'(x) < 0$ for any $x \in I$, then f is decreasing on I.

Concavity Test

If $f''(x) > 0$ for any $x \in I$, then f is concave upward on I.

If $f''(x) < 0$ for any $x \in I$, then f is concave downward on I.

Definition2 A **point of inflection** is a point where the graph has a tangent and concavity changes from upward to downward or vice versa. At such a point $f'' = 0$ or f'' is undefined.

The First Derivative Test

Suppose that c is a critical number of a continuous function f.

a) If f' changes from positive to negative at c, then f has a local maximum at c.
b) If f' changes from negative to positive at c, then f has a local minimum at c.
c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.

The Second Derivative Test

Suppose f'' is continuous near c.

a) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.
b) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

How to Graph a Function

X	
f'	
f	
f''	

f -domain
- x- and y-intercepts
- end-behavior ($\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$)
- behavior near the points where the function is not defined

f' - zeros
- study the sign

f'' - zeros
- study the sign