Sections 4.1 & 4.2 – Theorems

The Extreme Value Theorem (4.1)

This theorem gives conditions under which a function is guaranteed to have extreme values.

Hypothesis:	f continuous on a closed interval $[a,b]$
Conclusion:	f attains both an absolute maximum and an absolute minimum in $[a,b]$.

Examples of functions that satisfy the hypothesis:

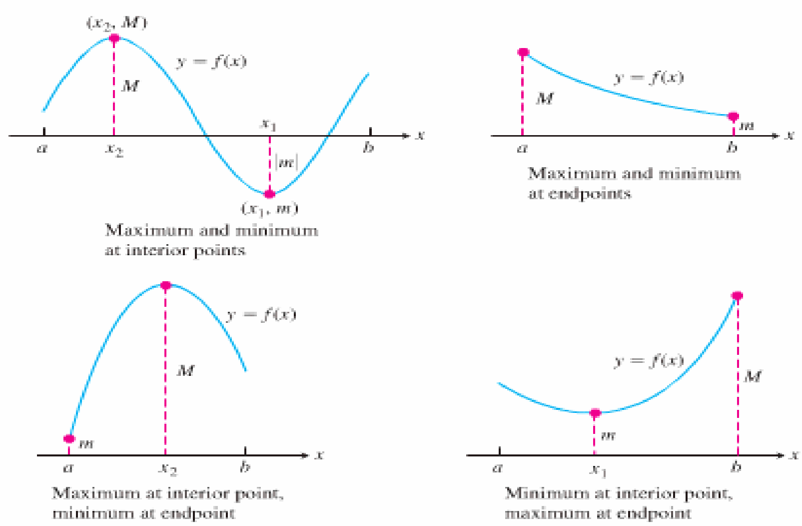


FIGURE 4.3 Some possibilities for a continuous function's maximum and minimum on a closed interval $[a, b]$.

Example of a function that does not satisfy the hypothesis:

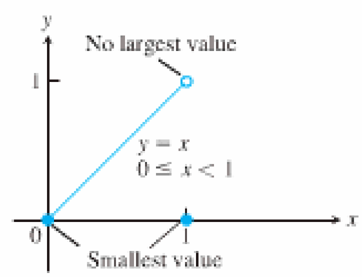


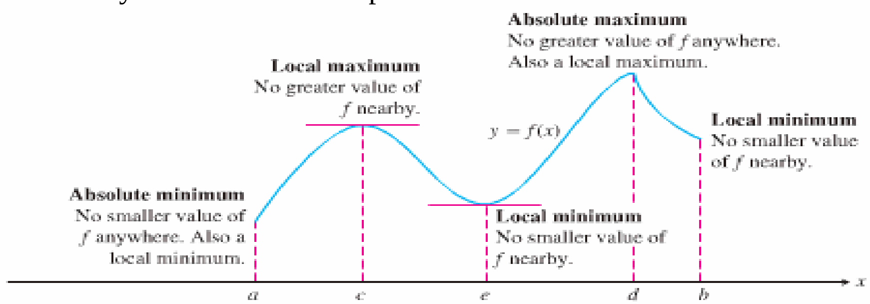
FIGURE 4.4 Even a single point of discontinuity can keep a function from having either a maximum or minimum value on a closed interval. The function

$$y = \begin{cases} x, & 0 \le x < 1 \\ 0, & x = 1 \end{cases}$$

is continuous at every point of $[0, 1]$ except $x = 1$, yet its graph over $[0, 1]$ does not have a highest point.

Fermat's Theorem (The First Derivative Theorem for Local Extreme Values) (4.1)

This theorem says that a function's derivative is always zero at an interior point where the function has a local extreme value and the derivative is defined.



Hypothesis:	f is a function c is an interior point of the domain of f f has a local minimum or maximum value at c $f'(c)$ exists
Conclusion:	$f'(c) = 0$

Rolle's Theorem (4.2)

This theorem says that between any two points where a differentiable function crosses a horizontal line there is at least one point on the curve where the tangent is horizontal.

Hypothesis:	f continuous on $[a,b]$ f differentiable on (a,b) $f(a) = f(b)$
Conclusion:	$\exists c \in (a,b)$ such that $f'(c) = 0$

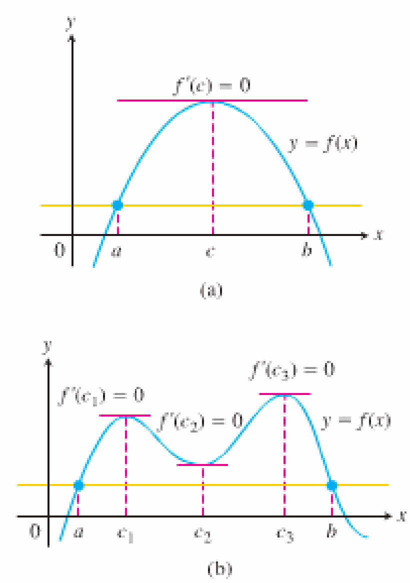
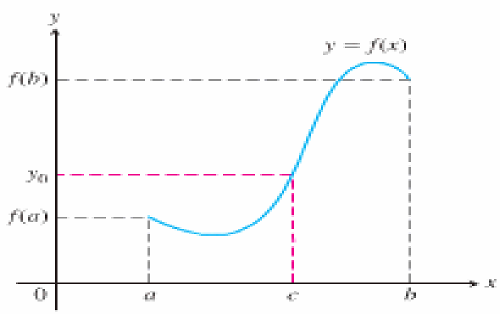


FIGURE 4.10 Rolle's Theorem says that a differentiable curve has at least one horizontal tangent between any two points where it crosses a horizontal line. It may have just one (a), or it may have more (b).

The Intermediate Value Theorem for Continuous Functions (2.6)

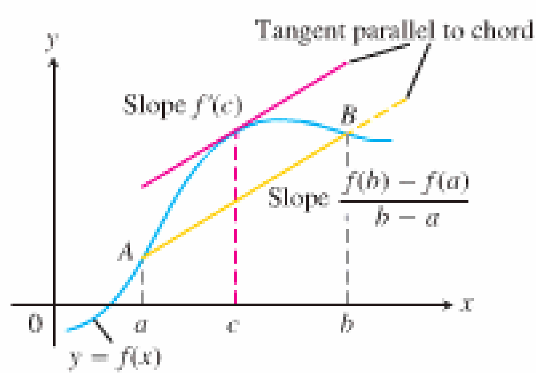
A function is said to have the Intermediate Value Property if whenever it takes on two values, it takes on all the values in between.

Hypothesis:	f continuous on a closed interval $[a,b]$
Conclusion:	f takes on every value between $f(a)$ and $f(b)$



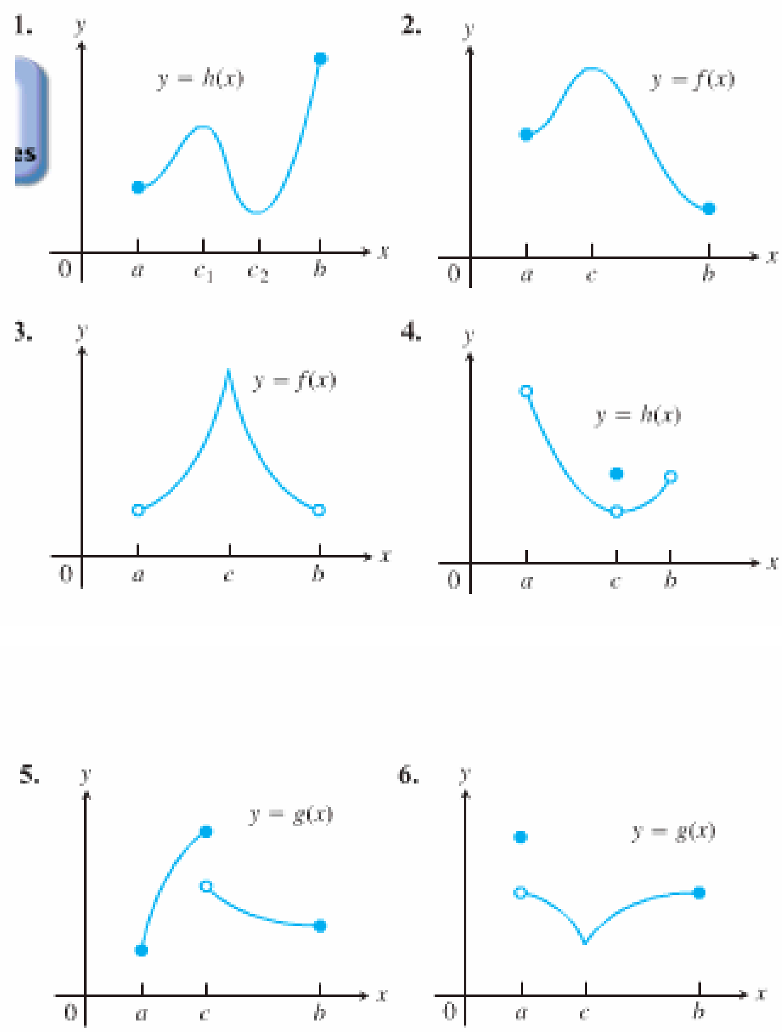
The Mean Value Theorem (4.2)
This theorem says that if a function is differentiable, then there is a point somewhere between A and B where the tangent line is parallel to the secant line AB.

Hypothesis:	f continuous on a closed interval $[a,b]$
	f differentiable on (a,b)
Conclusion:	$\exists c \in (a,b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b-a}$

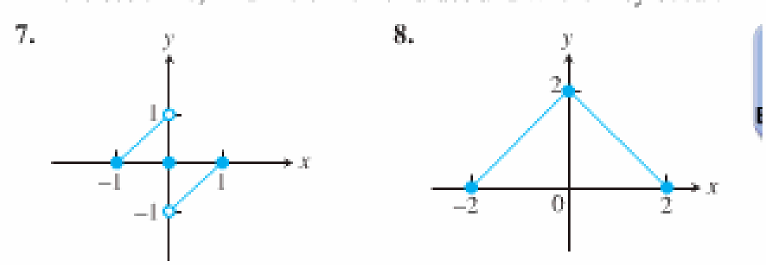


1. (4.1 - #1 – 8)

Find the extreme values and where they occur.



In Exercises 7–10, find the extreme values and where they occur.



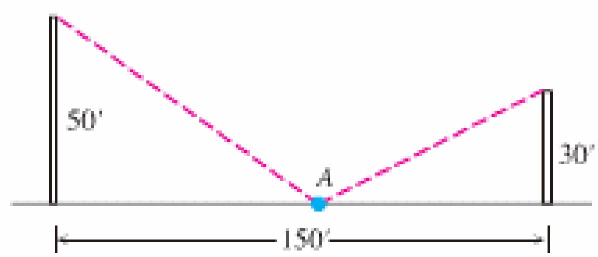
2. (4.1 - #71)

What is the largest possible area for a right triangle whose hypotenuse is 5 cm long?

3. (4.1 - # 68)

One tower is 50 ft high and another is 30 ft high. The towers are 150 ft apart. A guy wire is to run from point A to the top of each tower.

a) Locate point A so that the total length of guy wire is minimal.



4. (4.1 - # 65) Supertankers off-load oil at a docking facility 4 mi offshore. The nearest refinery is 9 mi east of the shore point nearest the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs $300,000 per mile if constructed underwater and $200,000 per mile if overland.

a) Locate point B to minimize the cost of the construction.