3.7 Rates of Change in the Natural and Social Sciences
3.8 Exponential Growth and Decay
3.9 Related Rates

Exercise 1 (3.7 - #8)

If a ball is thrown vertically upward with a velocity of 80ft/s, then its height after t seconds is $s(t) = 80t - 16t^2$.

a) What is the maximum height reach by the ball.

b) What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down?

Exercise 2 (3.7 - #14)

A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60cm/s. Find the rate at which the area within the circle is increasing after (a) 1 s, (b) 3s, and (c) 5 s. What can you conclude?

Exercise 3 (3.7 - #18)

If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Toricelli's Law gives the volume *V* of water remaining in the tank after *t* minutes as

$$V = 5000\left(1 - \frac{1}{40}t\right)^2, 0 \le t \le 40.$$

Find the rate at which water is draining from the tank after (a) 5 min, (b) 10min, (c) 20 min, (d) 40 min. At what time is the water flowing out the fastest? The slowest?

Exercise 4 (3.7 - #25)

A bacteria population triples every hour and starts with 400 bacteria. Find an expression for the number *n* of bacteria after *t* hours and use it to estimate the rate of growth of the bacteria population after 2.5 hours.

Exercise 5 (3.7 - #26)

The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function

$$n = f(t) = \frac{a}{1 + be^{-0.7t}}$$

where *t* is measured in hours. At time *t*=0 the population is 20 cells and is increasing at a rate of 12 cells/hour. Find the values of *a* and *b*. According to this model, what happens to the yeast population in the long run?

Exercise 6 (3.7 - #37)

The gas law for an ideal gas at absolute temperature *T* (in kelvins), pressure *P* (in atmospheres), and volume *V* (in lites) is $PV = nRT$, where *n* is the number of moles of the gas and $R = 0.0821$ is the gas constant. Suppose that, at a certain instant, *P* = 8.0 atm and is increasing at a rate of 0.10 atm/min and *V* = 10 L and is decreasing at a rate of 0.15 L/min. Find the rate of change of *T* with respect to time at that instant if *n* = 10 mol.

Exercise 7 (3.8 - # 2)

A common inhabitant of human intestines is the bacterium Escherichia coli, named after the German pediatrician Theodor Escherich, who identified it in 1885. A cell of this bacterium in a nutrient-broth medium divides into two cells every 20 minutes. The initial population of a culture is 50 cells.

a) Find the relative growth rate.
b) Find an expression for the number of cells after *t* hours.
c) Find the number of cells after 6 hours.
d) Find the rate of growth after 6 hours.
e) When will the population reach a million cells?

Exercise 8 (3.8 - #8)

Strontium-90 has a half-life of 28 days.
a) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after *t* days.
b) Find the mass remaining after 40 days.
c) How long does it take the sample to decay to a mass of 2 mg?
d) Sketch the graph of the mass function.

Exercise 9 (3.8 - # 14)

A curve passes through the point (0,5) and has the property that the slope of the curve at every point *P* is twice the *y*-coordinate of *P*. What is the equation of the curve?

Exercise 10 (3.9 - #2)

a) If *A* is the area of a circle with radius r and the circle expands as time passes, find *dA/dt* in terms of *dr/dt*.
b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 30 m?

Exercise 11 (3.9 - #4)

The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

Exercise 12 (3.9 - #5)

A cylindrical tank with radius 5 m is being filled with water at a rate of 3 $m^3/\min$. How fast is the height of the water increasing?

Exercise 13 (3.9 - #13)

A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

Exercise 14 (3.9 - #16)

At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

Exercise 15 (3.9 - #22)

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?

Exercise 16 (3. 9 - #24)

A particle moves along the curve $y = 2\sin\left(\frac{px}{2}\right)$. As the particle passes through the point $\left(\frac{1}{3}, 1\right)$ its x-coordinate increases at a rate of $\sqrt{10}$ cm/s. How fast is the distance from the particle to the origin changing at this instant?

Exercise 17 (3.9 - #25)

Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

Exercise 18 (3.9 - #26)

A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?

1) a) 100ft, b) 16ft/s, -16ft/s;

2) 7200p sq.cm/s, 216000p sq.cm/s, and 36000p sq.cm/s, the area grows at an increasing rate ;

3) -218.75 gal/min, -187.5 gal/min, -125 gal/min, 0 gal/min, water is lowing out the fastest when t=0, and slowest when t=40. As the tank empties, the water flows out more slowly;

4) 6850 bacteria/hr; 5) a=140, b=6, population stabilizes at 140 cells; 6) -0.2436 K/min;

7) a)about 207.9% per hour, b) $P = 50\left(8^t\right)$, c) 13,107,200 cells, d)27,255,656 cells per hour, e) 4.76 h;

8) a) $m(t) = 50 \cdot 2^{-t/28}$, b) 18.6 mg, c) 130 days;

9) $y = 5e^{2x}$; 10) $60p\, m^2/s$; 11) 140 cm^2/s; 12) $\frac{3}{25p}$ m/min;

13) $250\sqrt{3}$ mi/h; 14) $215/\sqrt{101}$ km/h; 15) $-\sqrt{65}/8$ m/s; 16) $1+3\sqrt{3}p/2$ cm/s;

17) $10{,}000 + \frac{800{,}000p}{9}$ cm^3/min ; 18) 4/5 ft/min