Section 5.1 – Estimating with Finite Sums

In this section we discover that in attempting to find the area under a curve or the distance traveled by a car, we end up with the same special type of limit.

The Area Problem

In trying to solve this problem we have to ask: What is the meaning of the word *area*? This question is easy to answer for regions with straight sides. For a rectangle, the area is defined as the product of the length and the width. The area of a triangle is half the base times the height. The area of a polygon is found by dividing it into triangles and adding the areas of the triangles.

However, it is not easy to find the area of a region with curved sides. We all have an intuitive idea of what the area of a region is. But part of the area problem is to make this intuitive idea precise by giving an exact definition of area.

Recall that in defining a tangent we first approximated the slope of the tangent line by slopes of secant line and then took the limit of these approximations. We pursue a familiar idea for areas. We first approximate the region S by rectangles and then we take the limit of the areas of these rectangles as we increase the number of rectangles.

The following example illustrates the procedure.

<u>Example 1</u>   a) Use four upper rectangles to estimate the area under the parabola $y = x^2$ from 0 to 1.
             b) Repeat a) using lower rectangles.

             c) Write a formula for the sum of the upper n rectangles and show that it approaches $\dfrac{1}{3}$.

             d) Repeat d) for lower rectangles.

The Distance Problem

Let's consider the distance problem: Find the distance traveled by an object during a certain time period if the velocity of the object is know at all times. If the velocity remains constant, then the distance problem is easy to solve by means of the formula

$$\text{Distance} = \text{velocity x time}$$

But if the velocity varies, it is not so easy to find the distance traveled.

<u>Example 2</u>   Suppose the odometer on our car is broken and we want to estimate the distance driven over a 30-second time interval. We take speedometer readings every five seconds and record them in the following table (in order to have the time and the velocity in consistent units, we first convert the velocity readings from mph to feet per second 1 mi/h = 5280/ 3600 ft/s):

Time (s)	0	5	10	15	20	25
Velocity (ft/sec)	25	31	35	43	47	46

For each time interval we assume that the velocity doesn't change much, so we can estimate the distance traveled during that time by assuming that the velocity is constant.

Estimate the total distance traveled by the car.

a) Use the velocity at the beginning of each time period as the assumed constant velocity.
b) Repeat the problem using the velocity at the end of each time period instead of the velocity at the beginning as the assumed constant velocity .
c) Sketch a graph of the velocity function of the car.