Section 4.7 – Optimization Problems

Problem 1 — You have been asked to design a 1-liter can shaped like a right circular cylinder. What dimensions will use the least material?

Problem 2 — Suppose that the revenue is $r(x) = 9x$ and the cost of producing the items is $c(x) = x^3 - 6x^2 + 15x$, where x represents thousands of units. Is there a production level that maximizes profit? If so, what is it?

Problem 3 (#4) — The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

Problem 4 (#6) — What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?

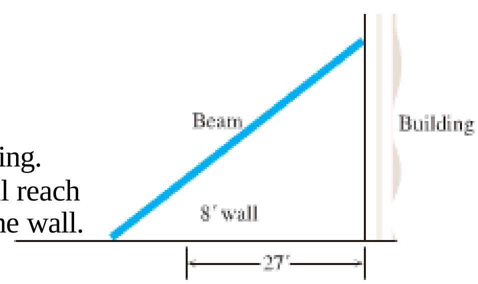
Problem 5 — A rectangle is to be inscribed under the arch of the curve $y = 4\cos(0.5x)$ from $x = -\pi$ to $x = \pi$. What are the dimensions of the rectangle with largest area, and what is the largest area?

Problem 6 — The height of an object moving vertically is given by

$$s = -16t^2 + 96t + 112,$$ with s in feet and t in seconds. Find

a) the object's initial velocity
b) its maximum height and when it occurs
c) its velocity when $s = 0$.

Problem 7 — The 8-ft wall shown here stands 27 ft from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.



Problem 8 — The position of two particles on the s-axis are $s_1 = \sin t$ and $s_2 = \sin\left(t + \frac{\pi}{3}\right)$, with s_1 and s_2 in meters and t in seconds.

a) At what time(s) in the interval $[0, 2\pi]$ do the particles meet?
b) What is the farthest apart that the particles ever get?
c) When in the interval $[0, 2\pi]$ is the distance between the particles changing the fastest?

Problem 9 — It costs you c dollars each to manufacture and distribute backpacks. If the backpacks sell at x dollars each, the number sold is given by

$$n = \frac{a}{x - c} + b(100 - x),$$

where a and b are positive constants. What selling price will bring a maximum profit?

Problem 10 (#10) The rate (in mg carbon/ m^3 / h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{100I}{I^2 + I + 4}$$

where I is the light intensity (measured in thousands of foot-candles). For what light intensity is P a maximum?

Problem 11 (#13) A farmer wants to fence in an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

Problem 12 (#22) Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3,0)$.

Problem 13 (#30) If two equal sides of an isosceles triangle have length a, find the length of the third side that maximizes the area of the triangle.

Problem 14 (#40) A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

Problem 15 (#48) A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM. At what time were the two boats closest together?

Problem 16 (#54) Find an equation of the line through the point (3,5) that cuts off the least area from the first quadrant.

Problem 17 (#56) At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

Problem 18 (#78) A painting in an art gallery has height h and is hung so that its lower edge is a distance d above the eye of an observer. How far from the wall should the observer stand to get the best view? (In other words, where should the observer stand so as to maximize the angle subtended at his eye by the painting?)