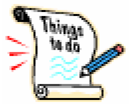
Section 4.5 – Applied Optimization Problems



Study Examples # 1, 3, and 5 from the textbook.

Problem 1 (Example 2) You have been asked to design a 1-liter can shaped like a right circular cylinder. What dimensions will use the least material?

Problem 2 (Example 5) Suppose that the revenue is $r(x) = 9x$ and the cost of producing the items is $c(x) = x^3 - 6x^2 + 15x$, where x represents thousands of units. Is there a production level that maximizes profit? If so, what is it?

Problem 3 (#4) A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

Problem 4 (#9) Your iron works has contracted to design and build a 500 cubic feet, square-based, open-top, rectangular steel holding tank for a paper company. The tank is to be made by welding thin stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible.

Problem 5 (#18) A rectangle is to be inscribed under the arch of the curve $y = 4\cos(0.5x)$ from $x = -p$ to $x = p$. What are the dimensions of the rectangle with largest area, and what is the largest area?

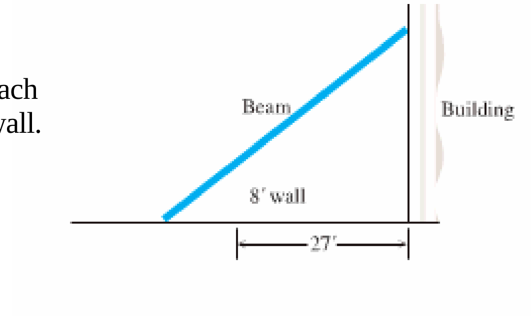
Problem 6 (#31) The height of an object moving vertically is given by

$s = -16t^2 + 96t + 112$, with s in feet and t in seconds. Find

a) the object's initial velocity
b) its maximum height and when it occurs
c) its velocity when $s = 0$.

Problem 7 (# 33)

The 8-ft wall shown here stands 27 ft from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.



Problem 8 (# 36)

The position of two particles on the *s*-axis are $s_1 = \sin t$ and $s_2 = \sin\left(t + \frac{p}{3}\right)$, with s_1 and s_2 in meters and t in seconds.

a) At what time(s) in the interval $[0, 2p]$ do the particles meet?

b) What is the farthest apart that the particles ever get?

c) When in the interval $[0, 2p]$ is the distance between the particles changing the fastest?

Problem 9 (#43)

It costs you c dollars each to manufacture and distribute backpacks. If the backpacks sell at x dollars each, the number sold is given by

$$n = \frac{a}{x - c} + b(100 - x),$$

where a and b are positive constants. What selling price will bring a maximum profit?

Problem 10 (#48)

Suppose that $c(x) = x^3 - 20x^2 + 20{,}000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items.