Section 3.1 – The Derivative of a Function
Graphing the Derivative

Exercises #27 – 30 / 3.1

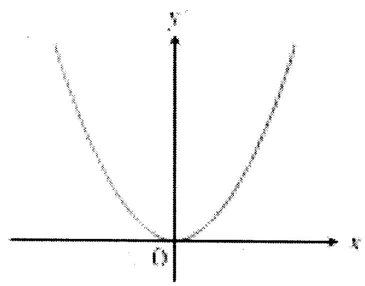
Match the functions graphed in #27 – 30 with the derivatives graphed in the accompanying figures a) – d).

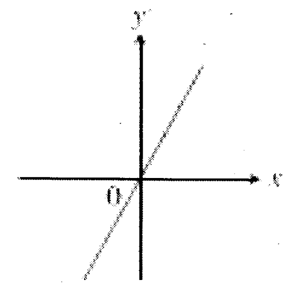
(27) when $x<0$, $f'(x)<0$ and increasing
when $x=0$, $f'(0)=0$
when $x>0$, $f'(x)>0$ and increasing
We see that f' is always increasing, so graph (b)

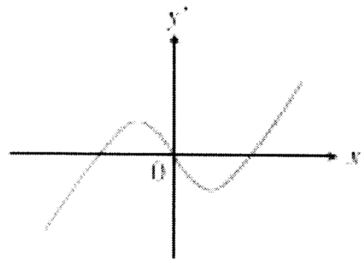
(28) when $x \neq 0$, $f'(x)>0$
when $x=0$, $f'(x)=0$
Also,
when $x<0$, f' is decreasing
when $x>0$, f' is increasing
so graph (a)

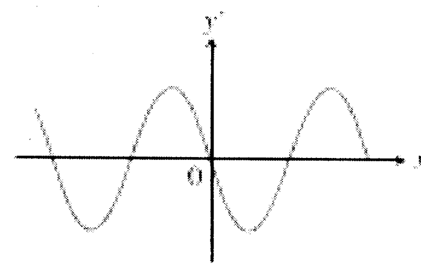
(29) $f'(x)=0$ for five x-values, so the graph of f' has 5 x-intercepts
so (d)

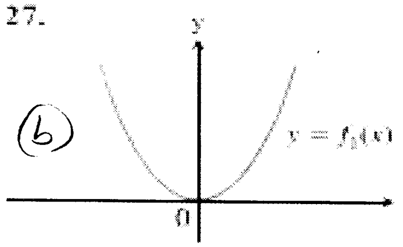
(30) $f'(x)=0$ 3 times so there are 3 x-intercepts for the graph of f'
so graph (c)

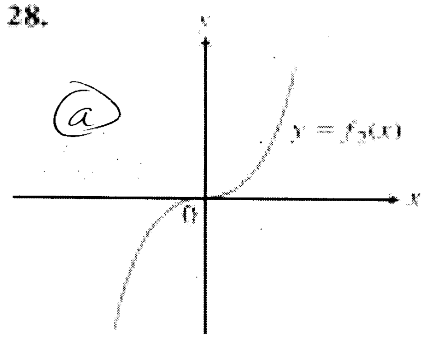

(a)

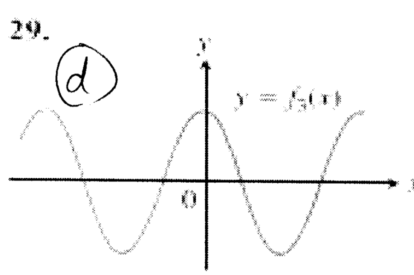

(b)

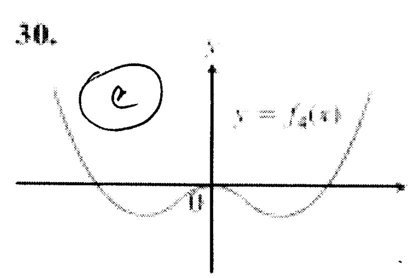

(c)


(d)

27.


28.


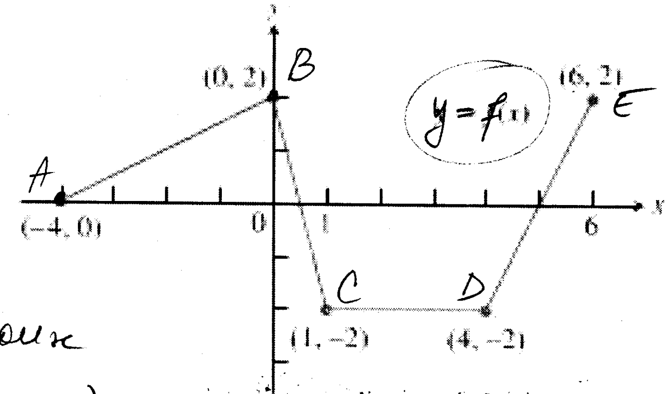
29.


30.


Exercise #31 / 3.1

a) The graph of the accompanying figure is made of line segments joined end to end. At which points of the interval $[-4,6]$ is f' not defined? Give reasons for your answer.

b) Graph the derivative of f.



Solution

(a) f' is not defined at $x=0$, $x=1$, $x=4$

At these points, the left-hand and right-hand derivatives are not equal:

$x=0$ $f'(0)$ doesn't exist because

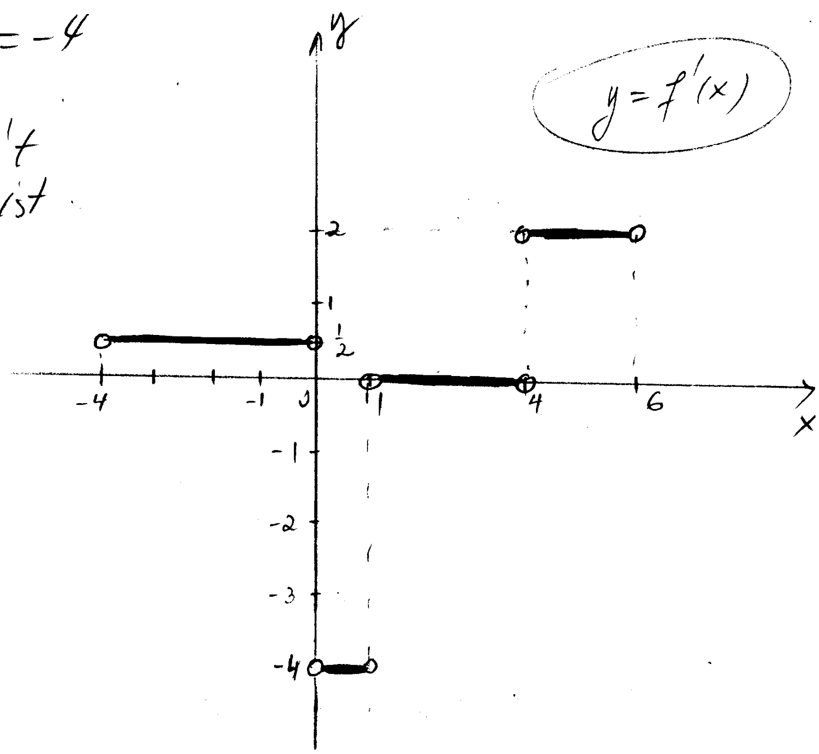
$$\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-}(\text{slope of } AB)$$
$$= \lim_{x\to 0^-}\frac{2}{4} = \frac{1}{2}$$

$$\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+}(\text{slope of } BC)$$
$$= \lim_{x\to 0^+}\frac{-4}{1} = -4$$

so $f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$ doesn't exist.

Repeat for $x=1$ and $x=4$

(b)



Note: The slope of a line is constant.

$m_{AB} = \frac{2}{4} = \frac{1}{2}$ $\qquad m_{CD} = 0$

$m_{BC} = \frac{-4}{1} = -4$ $\qquad m_{DE} = \frac{4}{2} = 2$

Exercises #35 , 38 / 3.1

Compare the right-hand and left-nad derivatives to show that the functions are not differentiable at the point P.

Solution

(35) $P(0,0)$

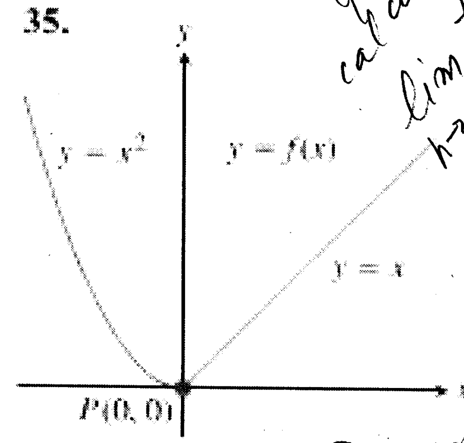
We'll show $\lim\limits_{x\to 0^-} \dfrac{f(x)-f(0)}{x-0} \neq \lim\limits_{x\to 0^+} \dfrac{f(x)-f(0)}{x-0}$

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{x^2-0}{x} = \lim_{x\to 0^-} x = 0$$

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{x-0}{x} = \lim_{x\to 0^+} 1 = 1$$

so $f'(0)$ doesn't exist.

35.

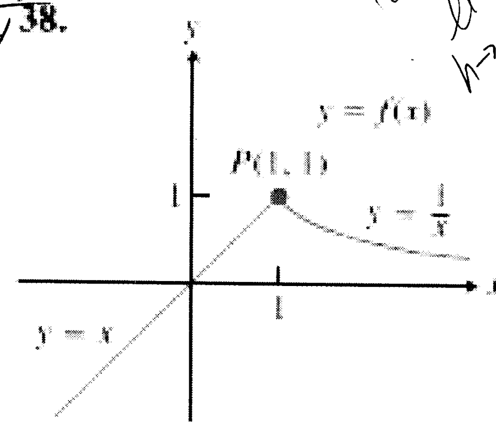


Note) You could also calculate $\lim\limits_{h\to 0} \dfrac{f(h)-f(0)}{h}$

(38) $P(1,1)$

We'll show $\lim\limits_{x\to 1^-} \dfrac{f(x)-f(1)}{x-1} \neq \lim\limits_{x\to 1^+} \dfrac{f(x)-f(1)}{x-1}$

38.



Note) You could also calculate $\lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$

$$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{x-1}{x-1} = \lim_{x\to 1^-} 1 = 1$$

$$\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{\frac{1}{x}-1}{x-1} = \lim_{x\to 1^+} \frac{\frac{1-x}{x}}{x-1} = \lim_{x\to 1^+} \frac{1-x}{x(x-1)} = \lim_{x\to 1^+} \frac{-1}{x} = \frac{-1}{1} = -1$$

so $f'(1)$ doesn't exist