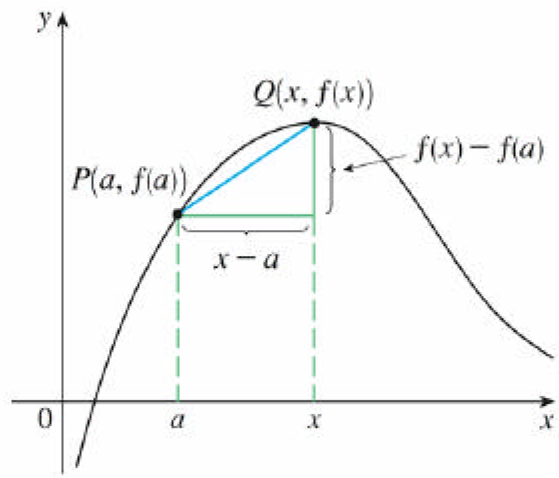
## Section 2.7
## Derivatives and Rates of Change – Part I

The problem of finding the tangent line to a curve and the problem of finding the velocity of an object both involve finding the same type of limit, as we saw in 2.1. This special type of limit is called a derivative and we will see that it can be interpreted as a rate of change in any of the natural or social sciences or engineering.

**Tangents**

How did you define the tangent to the graph $y = f(x)$ at the point $P(a, f(a))$?




<u>Definition 1</u> The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through $P$ with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

<u>Definition2</u> We say that the curve $y = f(x)$ ha s a **vertical tangent** at $(a, f(a))$ if and only if the slope of the tangent at that point is $\infty$ or $-\infty$.

**Exercise 1** Find the equation of the tangent line to the parabola $y = x^2$ at the point $P(2,4)$.

**Exercise 2** Find the equation of the tangent line to $y = \frac{1}{\sqrt{x}}$ at the point $(1,1)$.

**Exercise 3** a) Find the slope of the curve $y = \frac{1}{x}$ at any point $x = a \neq 0$. What is the slope at the point $x = -1$?

b) Where does the slope equal $-\frac{1}{4}$?

c) What happens to the tangent to the curve at the point $\left(a, \frac{1}{a}\right)$ as $a$ changes?

**Exercise 4** The equation $f(t)=16t^2$ gives the distance (in ft) of a rock falling freely during the first $t$ seconds.

a) Find the average speed of the rock between $t=1$ and $t=3$ seconds.
b) Find the instantaneous velocity at exactly 1 second.

**Exercise 5** Let $f(x)=\dfrac{x-1}{x+1}$. Find the slope of the tangent at $x=0$.

**Exercise 6** Let $f(x)=x^2+4x-1$. At what points does the graph have a horizontal tangent?

**Exercise 7** Find an equation of the line having slope $\dfrac{1}{4}$ that is tangent to $f(x)=\sqrt{x}$.

**Exercise 8** Does the graph of $f(x)=\begin{cases} x^2\sin\dfrac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ have a tangent at the origin?

**Exercise 9** Does the graph of $y=x^{\frac{1}{3}}$ have a vertical tangent at the origin?

**Exercise 10** Does the graph of $f(x)=\begin{cases} 0, & x<0 \\ 1, & x\geq 0 \end{cases}$ have a vertical tangent at $(0,1)$?

Answers: 1) $y=4x-4$; 2) $y=-\dfrac{1}{2}x+\dfrac{3}{2}$; 3) a) $-\dfrac{1}{a^2}$; b) $\left(2,\dfrac{1}{2}\right)$ and $\left(-2,-\dfrac{1}{2}\right)$; 4) 64 ft/sec; 32 ft/sec; 5) 2; 6) $(-2,-5)$; 7) $y=\dfrac{1}{4}x+1$; 8) yes, horizontal; 9) yes; 10) no.

Solutions