Chapter 1 – Review
Functions

1. Let $f(x)=7-3x$. Answer the following questions:
   a) What type of function is this?
   b) Graph the function.
   c) Find the domain and range.
   d) Find the slope of the line.
   e) Is this function increasing or decreasing?
   f) Is this function even , odd, or neither?
   g) Find an equation for the line passing through $(1,-3)$ that is perpendicular to $f(x)=7-3x$.

2. Let $g(x)=x$. Answer the following questions:
   Repeat questions a) – f) from above.
   g) Find a formula to shift the graph up 3 units.
   h) Find a formula to stretch the graph vertically by a factor of 2.
   i) Find a formula to compress the graph vertically by a factor of 2.
   j) Compute $(f\circ g)(x)$ and $g(f(x))$ for $f(x)=7-3x$ and $g(x)=x+3$.

3. Let $h(x)=x^2$. Answer the following questions:
   Repeat questions a), b), c), e), f) from above.
   g) Find a formula to shift the graph to the right 1 unit.
   h) Find a formula to shift the graph to the left 2 units.
   i) Find a formula to compress the graph horizontally by a factor of 2.

4. Let $f(x)=x^3, g(x)=\frac{1}{x}, h(x)=\frac{1}{x^2}, l(x)=\sqrt{x}$.
   Repeat questions a), b), c), e), f) from above.
   g) Find a formula to compress the graph of $l(x)=\sqrt{x}$ vertically by a factor of 2 followed by a reflection about the x-axis.
   h) Find and simplify $\frac{g(x+h)-g(x)}{h}$ for $h\neq 0$.

5. Draw a graph of each function and state its domain and range. State the intervals on which each function is increasing, decreasing, or constant.

   a) $f(x)=\begin{cases} 2x-1, & -3<x<2 \\ -3, & 2\le x<4 \\ x^{\frac{1}{2}}, & x\ge 4 \end{cases}$

   b) $f(x)=\sqrt{4-x^2}$

c) $f(x)=\sqrt{x^2-4}$

d) $f(x)=3^x$

e) $f(x)=2e^{-x}+3$

f) $f(x)=\log_2 x$

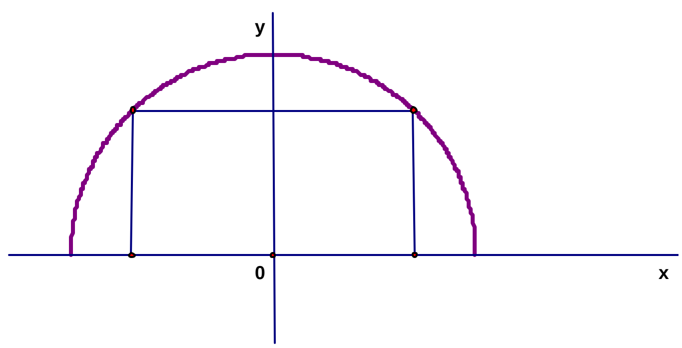
g) $f(x)=\ln x$

6. Let $f(x)=\sin 3x-\frac{1}{2}$. Answer the following questions:
   a) Graph the function over one period.
   b) What is the domain and range?
   c) Find the exact x-intercepts from the graph shown.

7. The amount A (in grams) of a radioactive material remaining after t days is given by $A=270e^{-0.025t}$.
   a) How many grams of material were there initially?
   b) How many grams remain after 8 days?
   c) When will only 100 grams remain?

8. Use properties of logs to expand $\ln\left(\frac{x^3yz^2}{t^5}\right)$.

9. A rectangle is inscribed inside $f(x)=\sqrt{9-x^2}$ as shown.
   a) Express the area of the rectangle as a function of x.
   b) What is the implied domain for x ?



Solving Equations

Definition The standard form of a quadratic or second degree equation in one variable is $ax^2 + bx + c = 0$ where $a, b, c \in \mathbb{R}; a \neq 0$.

Solving quadratic equations

(1) THE FACTORING METHOD – used to solve equations of the form $ax^2 + bx + c = 0$ that are factorable (see factoring methods on page 2)

Zero-Factor Property: The product of two factors equals zero if and only if one of the factors (or both) is zero.

$$AB = 0 \Leftrightarrow A = 0 \ \ or \ \ B = 0$$

(2) EXTRACTION OF ROOTS – used to solve equations of the form

$$x^2 = k \qquad \text{or} \qquad (x-p)^2 = k.$$

$$\sqrt{x^2} = \sqrt{k} \qquad\qquad \sqrt{(x-p)^2} = \sqrt{k}$$

$$x = \pm\sqrt{k} \qquad\qquad x - p = \pm\sqrt{k}$$

$$x = p \pm \sqrt{k}$$

(3) COMPLETING THE SQUARE $ax^2 + bx + c = 0$

Step 1: Coefficient of x^2 equal to 1.

Step 2: Constant isolated.

Step 3: Complete the square by adding $\left(\frac{1}{2} \cdot coefficient \ of \ x\right)^2$ to both sides of the equation and solve by the extraction of roots method.

(4) QUADRATIC FORMULA If $ax^2 + bx + c = 0$, then the solutions are given by:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Definition</u> The discriminant of a quadratic equation is $\Delta = b^2 - 4ac$

<u>Properties</u> **(1)** If $a, b, c \in \mathbb{R}$, then:

If $\Delta > 0$, the equation has two distinct real solutions.

If $\Delta = 0$, the equation has one real (rational) solution.

If $\Delta < 0$, the equation has two complex (nonreal) solutions.

(2) If $a, b, c \in \mathbb{Q}$, then:

If Δ is a perfect square, the equation has **rational solutions**.

If Δ is not a perfect square, then the equation has **irrational solutions.**

Factoring a polynomial

1. GCF Factor out the greatest common factor (if any).

2. Special products

<u>Two terms</u>

$$a^2 - b^2 = (a-b)(a+b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

<u>Three terms</u>

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

3. Factoring technique to factor out a trinomial $ax^2 + bx + c$

<u>$a = 1$</u>

$$x^2 + bx + c = (x + \square)(x + \square)$$

product = c
sum = b

<u>$a \neq 1$</u>

split the middle term bx

$$ax^2 + bx + c = ax^2 + \square x + \square x + c$$

product = ac
sum = b
then factor by grouping

4. If more than four term, factor by grouping.