Math 160

Homework # 3

Solutions to selected problems

Review page 275

(18) $f(x)=x^5-4x^4-3x^3+34x^2-52x+24$

$$\begin{array}{c|cccccc} & 1 & -4 & -3 & 34 & -52 & 24 \\ \hline 2 & 1 & -2 & -7 & 20 & -12 & 0 \end{array}$$

$f(x)=(x-2)(x^4-2x^3-7x^2+20x-12)$

$$\begin{array}{c|ccccc} & 1 & -2 & -7 & 20 & -12 \\ \hline 2 & 1 & 0 & -7 & 6 & 0 \end{array}$$

$f(x)=(x-2)(x-2)(x^3-7x+6)$

$f(x)=(x-2)^2(x^3-7x+6)$

$$\begin{array}{c|cccc} & 1 & 0 & -7 & 6 \\ \hline 2 & 1 & 2 & -3 & 0 \end{array}$$

$f(x)=(x-2)^2(x-2)(x^2+2x-3)$

$\boxed{f(x)=(x-2)^3(x+3)(x-1)}$

(20) $f(x)=x^6+2x^4+x^2$

$f(x)=x^2(x^4+2x^2+1)$

$f(x)=x^2(x^2+1)^2$

zeros: $\boxed{x=0}$ $m=2$

$x^2+1=0$, $x^2=-1$

$\boxed{x=\pm i}$ $m=2$

(24)

$x^4+9x^3+31x^2+49x+30=0$

Possible rational zeros:

$\frac{p}{q}=\frac{\text{factor of } 30}{\text{factor of } 1}$

$=\frac{\pm1,\pm2,\pm3,\pm5,\pm6,\overset{\pm10}{\vee}\pm15,\pm30}{\pm1}$

$\frac{p}{q}\in\{\pm1,\pm2,\pm3,\pm5,\pm6,\overset{\pm10}{\vee}\pm15,\pm30\}$

Note that $f(1)\neq0$, $f(-1)\neq0$

Try $x=-2$

$$\begin{array}{c|ccccc} & 1 & 9 & 31 & 49 & 30 \\ \hline -2 & 1 & 7 & 17 & 15 & 0 \end{array}$$

$f(x)=(x+2)(\underbrace{x^3+7x^2+17x+15})$

possible rational zeros

$\{\pm1,\pm3,\pm5,\pm15\}$

Try $x=-3$

$$\begin{array}{c|cccc} & 1 & 7 & 17 & 15 \\ \hline -3 & 1 & 4 & 5 & 0 \end{array}$$

$f(x)=(x+2)(x+3)(x^2+4x+5)$

The solutions are:

$\boxed{\begin{array}{l} x=-2 \\ x=-3 \end{array}}$

$x^2+4x+5=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-4\pm\sqrt{16-4(5)}}{2}$

$x=\frac{-4\pm\sqrt{-4}}{2}=\frac{-4\pm2i}{2}$

$\boxed{x=-2\pm i}$

(33) Graph

$$f(x) = \frac{x}{(x+5)(x^2-5x+4)}$$

$$f(x) = \frac{x}{(x+5)(x-4)(x-1)}$$

Domain: $x \in \mathbb{R} \setminus \{-5, 1, 4\}$

V.A. $x=-5,\ x=1,\ x=4$

H.A. $y=0$

x-n: $(0,0)$

Test points:

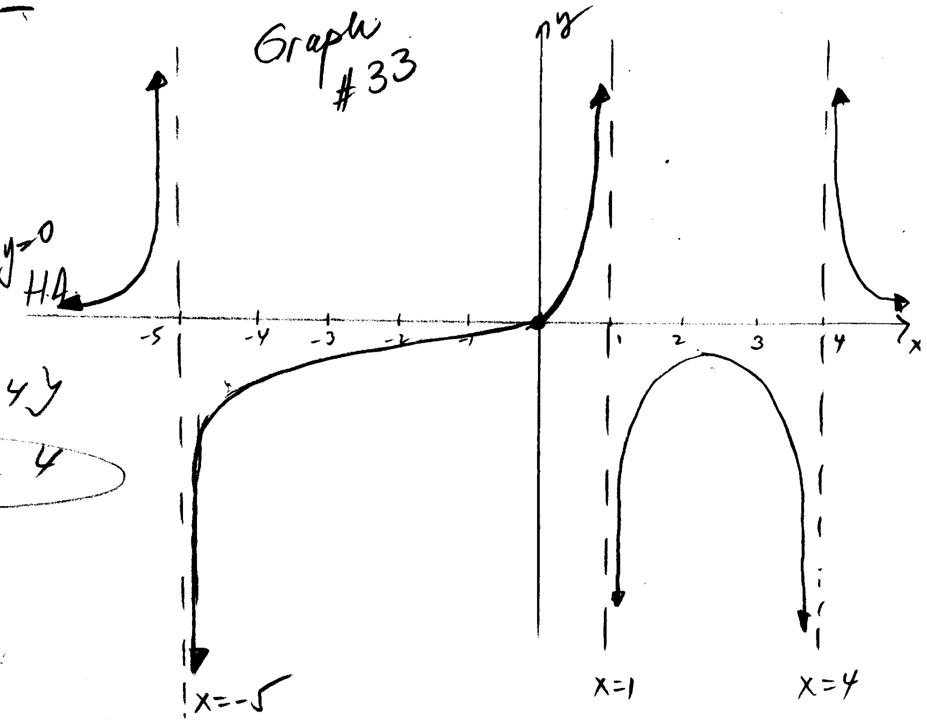
$x=-6,\ f(-6) = \frac{-6}{(-1)(-10)(-7)} > 0$

$x=-3\quad f(-3) = \frac{-3}{(2)(-7)(-4)} < 0$

$x=0.5\quad f(0.5) = \frac{0.5}{(5.5)(-3.5)(-0.5)} > 0$

$x=2\quad f(2) = \frac{2}{7(-2)(1)} < 0$

$x=5\quad f(5) = \frac{5}{10(1)(4)} > 0$



---

(34) $f(x) = \frac{x^3-2x^2-8x}{-x^2+2x}$

$$f(x) = \frac{x(x^2-2x-8)}{x(-x+2)}$$

$$f(x) = \frac{\cancel{x}(x-4)(x+2)}{\cancel{x}(2-x)}$$

Domain: $x \in \mathbb{R} \setminus \{0, 2\}$

$$f(x) = \frac{(x-4)(x+2)}{2-x}$$

Note: bc $x=0$, hole at $(0,-4)$

V.A. $x=2$

H.A. none

O.A Divide x^2-2x-8 by $-x+2$

$$\begin{array}{r|l} & -x \\ \hline -x+2 & x^2-2x-8 \\ & -x^2+2x \\ \hline & -8\ R \end{array}$$

$y=-x$ O.A.

x-n: $y=0$ iff $x=-2,\ x=4$

y-n: none $(x \neq 0)$

when $x=0,\ y=-4$

Hole at $(0,-4)$

intersection of graph with

O.A. $y=-x$

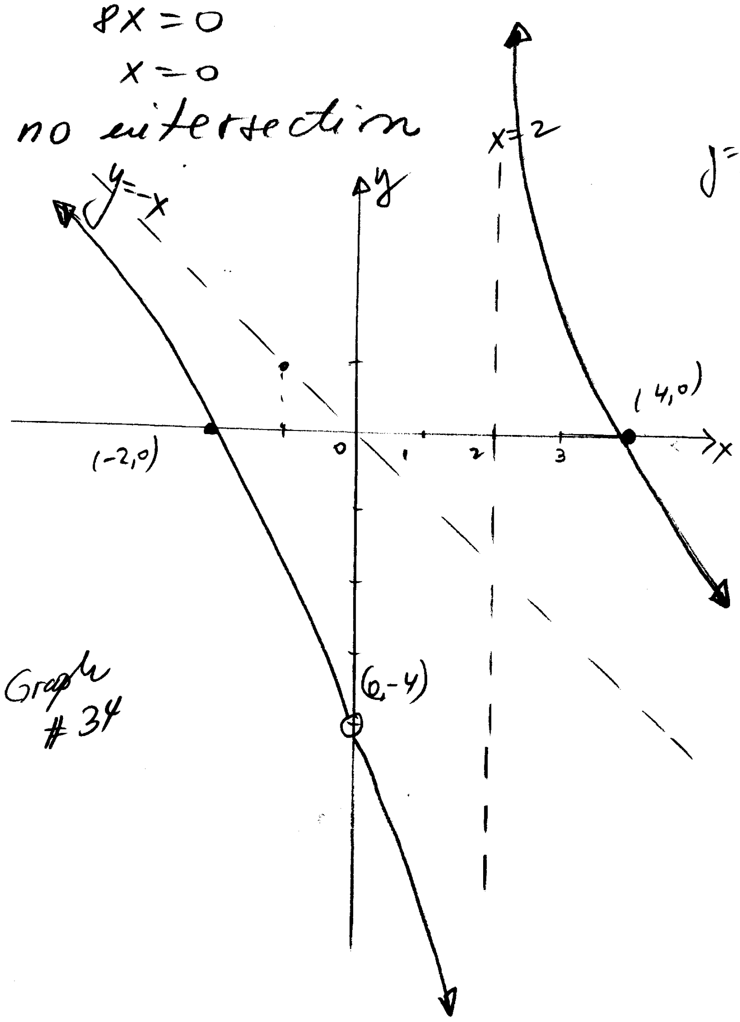
$$\frac{x^3-2x^2-8x}{-x^2+2x} = -x$$

$x^3-2x^2-8x = x^3-2x^2$

$8x = 0$

$x = 0$

no intersection



Graph #34

---

(36) $f(x) = \dfrac{3x^2+x-10}{x^2+2x}$

$f(x) = \dfrac{(3x-5)\cancel{(x+2)}}{x\cancel{(x+2)}} = \dfrac{3x-5}{x}$

Domain: $x \in \mathbb{R} \setminus \{0,-2\}$

V.A. $x = 0$

H.A. $y = 3$

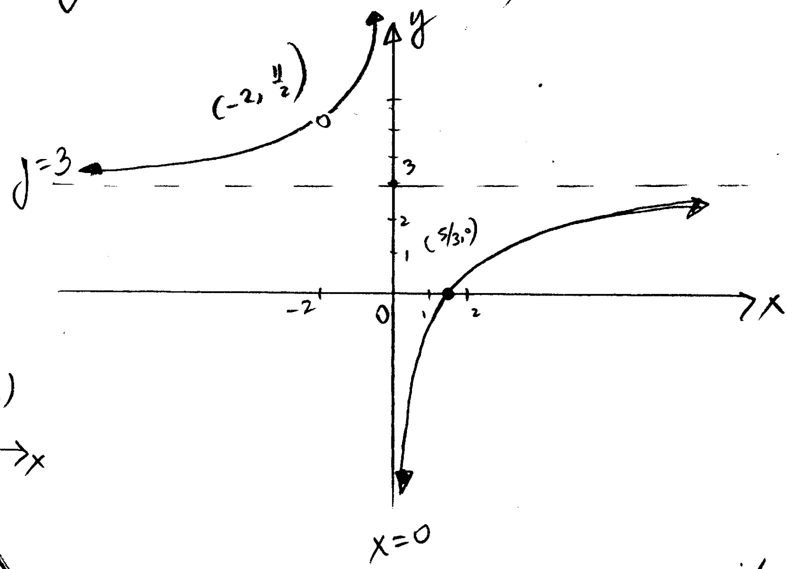
Note: hole at $(-2, \frac{11}{2})$

x-n: $y=0$ iff $3x-5=0$

$x = \frac{5}{3}$

$(\frac{5}{3}, 0)$

y-n: none $(x \neq 0)$



Check intersection of graph with H.A. $y=3$

$\dfrac{3x^2+x-10}{x^2+2x} = 3$

$\cancel{3x^2}+x-10 = \cancel{3x^2}+6x$

$5x = -10$

$x = -2$ not possible

(OR $\dfrac{3x-5}{x} = 3$

$3x-5 = 3x$

$-5 = 0$ false)

thus, no intersection

(37) $f(x) = \dfrac{-2x^2 - 8x - 6}{x^2 - 6x + 8}$

$f(x) = \dfrac{-2(x^2 + 4x + 3)}{(x-2)(x-4)}$

$f(x) = \dfrac{-2(x+1)(x+3)}{(x-2)(x-4)}$

Domain: $x \in \mathbb{R} \setminus \{2, 4\}$

V.A. $x = 2,\ x = 4$

H.A. $y = -2$

x-in: $y = 0$ iff $x = -1,\ x = -3$

y-in: $x = 0$, $y = \dfrac{-2(1)(3)}{(-2)(-4)} = \dfrac{-3}{4}$

$\left(0, \frac{-3}{4}\right)$

check intersection of graph with H.A. $y = -2$

$\dfrac{-2x^2 - 8x - 6}{x^2 - 6x + 8} = -2$

$\cancel{-2x^2} - 8x - 6 = \cancel{-2x^2} + 12x - 16$

$-6 + 16 = 12x + 8x$

$20x = 10$

$x = \frac{1}{2}$

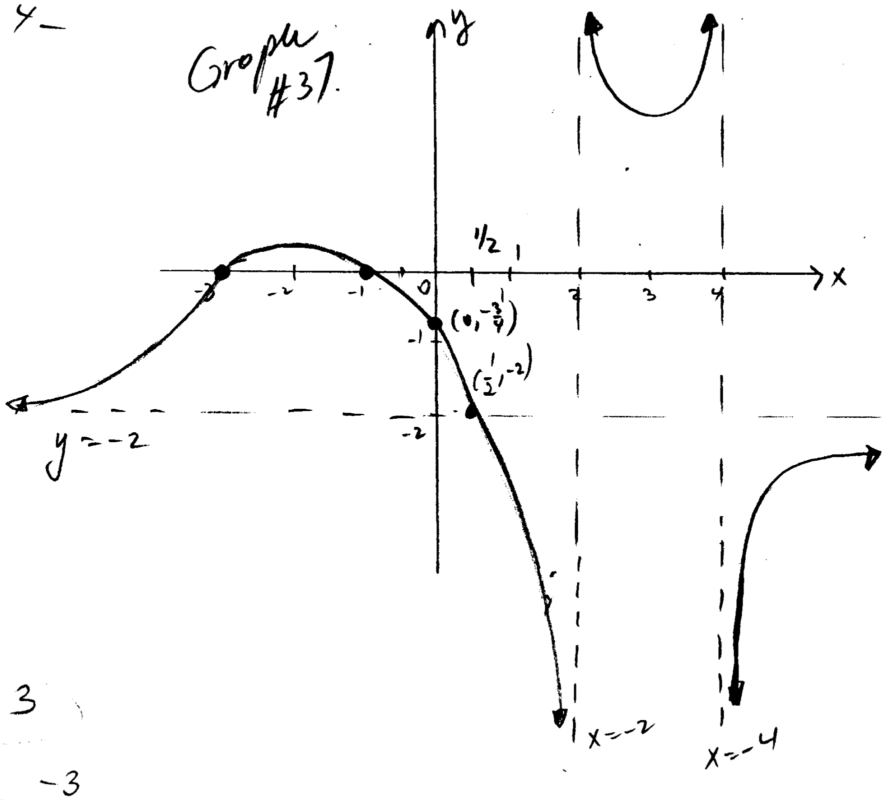
if $x = \frac{1}{2}$, $y = -2$ $\left(\frac{1}{2}, -2\right)$

Test points:

$x = 3$, $f(3) = \dfrac{-2(4)(6)}{1(-1)} > 0$

$x = 5$, $f(5) = \dfrac{-2(6)(8)}{(3)(1)} < 0$

Graph #37.



SECTION 4.1

(14) $f(x) = 2x^3 - 4$

Let $f(x_1) = f(x_2)$

then $2x_1^3 - 4 = 2x_2^3 - 4$

$\Rightarrow 2x_1^3 = 2x_2^3$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

So f is one-to-one

㉔ $f(x)=\frac{2x-7}{9x+1}$

Domain of f: $9x+1\neq 0$

$x\neq -\frac{1}{9}$

Range of f:

let $y=\frac{2x-7}{9x+1}$

$y(9x+1)=2x-7$

$9xy+y=2x-7$

$y+7=2x-9xy$

$y+7=x(2-9y)$

$x=\frac{y+7}{2-9y}\ ,\ y\neq\frac{2}{9}$

Therefore,

$f:\mathbb{R}\setminus\{-\frac{1}{9}\}\to\mathbb{R}\setminus\{\frac{2}{9}\}$

$f^{-1}:\mathbb{R}\setminus\{\frac{2}{9}\}\to\mathbb{R}\setminus\{-\frac{1}{9}\}$

OR, to find Range of f

Note that H.A. $y=\frac{2}{9}$ for the graph of f

Also, note that

$\frac{2x-7}{9x+1}\neq\frac{2}{9}$

(if $\frac{2x-7}{9x+1}=\frac{2}{9}$, then

$9(2x-7)=2(9x+1)$

$18x-63=18x+2$

$-63=2$ false)

therefore, $y\neq\frac{2}{9}$

(60) ⓑ Determine a linear function that models these data

Use two arbitrary points:

$(1950, 2773)$ and $(2000, 12,717)$

$m=\frac{12717-2773}{2000-1950}=198.88$

$y-y_1=m(x-x_1)$

$y-12717=198.88(x-2000)$

$\boxed{f(x)=198.88x-385,043}$

ⓒ let $y=198.88x-385,043$ and solve for x

$x=\frac{y+385,043}{198.88}$

$\boxed{f^{-1}(y)=\frac{y+385,043}{198.88}=}\ x$

x = year

y = number of radio stations

$f(x)=y \iff f^{-1}(y)=x$

Thus, $f^{-1}(y)$ gives the year when x radio stations were on the air

SECTION 4.2

(10) Solve

$9^{2x}\left(\frac{1}{3}\right)^{x+2} = 27\cdot\left(3^{x}\right)^{-2}$

$\left(3^{2}\right)^{2x}\left(3^{-1}\right)^{x+2} = 3^{3}\left(3^{-2x}\right)$

$3^{4x}\,3^{-x-2} = 3^{3-2x}$

$3^{4x-x-2} = 3^{3-2x}$

$3^{3x-2} = 3^{3-2x}$

The exponential function is one-to-one $\Rightarrow$

$3x-2 = 3-2x$

$5x=5$, $\boxed{x=1}$

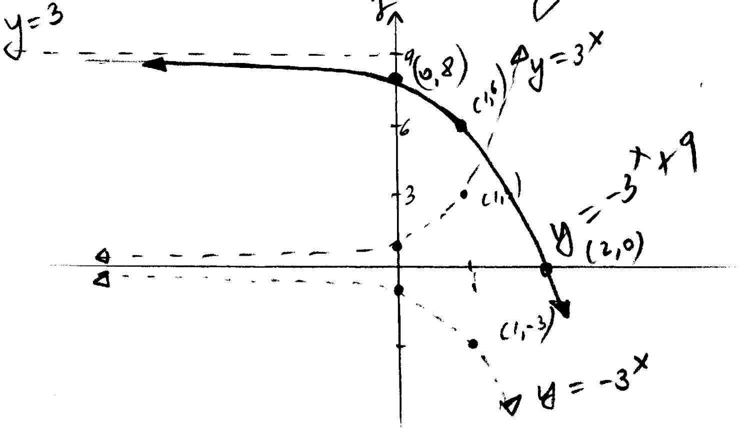
(18) Graph $f(x) = -3^{x}+9$

1st $y=3^{x}$ H.A. $y=0$

2nd $y=-3^{x}$ reflection about x-axis

3rd $y=-3^{x}+9$ shift up 9 units

H.A. $y=9$



Note that $x \in \mathbb{R}$

$y \in (-\infty, 9)$

y-int: $(0,8)$.

x-int: $-3^{x}+9=0$

$3^{x}=9$

$x=2$ $(2,0)$