Math 160

Solutions

4.3

(13) $f(x) = xe^x + e^x$

$f(x) = 0$

$xe^x + e^x = 0$

$e^x(x+1) = 0$

$e^x \neq 0$ $\Rightarrow$ $x+1=0$

$\boxed{x=-1}$

4.4

(31) Solve:

$e^{2\ln x} = 9$

Condition: $x > 0$

$e^{\ln x^2} = 9$ $\quad (b^{\log_b x} = x)$

$x^2 = 9$

$x = \pm 3$, but $x>0$ $\Rightarrow$ $\boxed{x=3}$

(34) Solve: $e^{x\ln 2} = 0.25$

$e^{\ln 2^x} = 0.25$

$2^x = \frac{1}{4}$

$2^x = 2^{-2}$

The exp. function is one-to-one $\Rightarrow$ $\boxed{x=-2}$

OR $\left(e^{x\ln 2} = \left(e^{\ln 2}\right)^x = 2^x \right)$

(55) $q = q_0 (2)^{-\frac{t}{1600}}$

Solve for t.

$2^{-\frac{t}{1600}} = \frac{q}{q_0}$

$\log_2\left(2^{-\frac{t}{1600}}\right) = \log_2\left(\frac{q}{q_0}\right)$

$-\frac{t}{1600} = \log_2\left(\frac{q}{q_0}\right)$

$\boxed{t = -1600\log_2\left(\frac{q}{q_0}\right)}$

(65) $\ln W = \ln 2.4 + (1.84)h$

h = height (in m)

W = weight (in kg)

(a) $\ln W = \ln 2.4 + (1.84)h$

$\Leftrightarrow$

$W = e^{(\ln 2.4 + 1.84h)}$

$W = e^{\ln 2.4} \cdot e^{1.84h}$

$\boxed{W = (2.4)e^{1.84h}}$

(b) $W = ?$ if $h = 1.5\,m$

$W = (2.4)e^{(1.84)1.5}$

$W \approx 37.92$ kg

(66) $A = Pe^{rt}$

P = principal
r = annual interest rate
t = time (in yr)
A = amount in account

$A = 25{,}000$
$P = 6000$
$r = 0.06$
$t = ?$

$$25{,}000 = 6000\, e^{0.06t}$$

$$e^{0.06t} = \frac{25000}{6000}$$

$$e^{0.06t} = \frac{25}{6} \quad / \ln$$

$$\ln e^{0.06t} = \ln\left(\frac{25}{6}\right)$$

$$0.06t = \ln\left(\frac{25}{6}\right)$$

$$\boxed{t = \frac{\ln\left(\frac{25}{6}\right)}{0.06}} \qquad t \approx 23.8\text{ yr.}$$

(68) $\log P = a + \dfrac{b}{c+T}$

Find P.

$$\log P = a + \frac{b}{c+T} \iff$$

$$\boxed{P = 10^{a + \frac{b}{c+T}}} = \boxed{10^{a} \cdot 10^{\frac{b}{c+T}}}$$

(def. of log)

(74) $A(t) = A_0 e^{-0.0239t}$

A_0 = amount currently in field
t = time (in years)

The field is currently 2.5 times the safe level S $\Rightarrow$

set $A_0 = 2.5S$
$A(t) = S$

$$\cancel{S} = 2.5\,\cancel{S}\, e^{-0.0239t}$$

$$e^{-0.0239t} = \frac{1}{2.5} \quad / \ln$$

$$\ln e^{-0.0239t} = \ln(0.4)$$

$$-0.0239t = \ln 0.4$$

$$\boxed{t = \frac{\ln 0.4}{-0.0239} \approx 38.3\text{ yr.}}$$

$\boxed{4.5}$

(8) Expand

$$\ln\left(x\sqrt[3]{\frac{y^4}{z^5}}\right) = \ln x + \ln\sqrt[3]{\frac{y^4}{z^5}}$$

$$= \ln x + \ln\left(\frac{y^4}{z^5}\right)^{\frac{1}{3}}$$

$$= \ln x + \frac{1}{3}\ln\left(\frac{y^4}{z^5}\right)$$

$$= \ln x + \frac{1}{3}\left(\ln y^4 - \ln z^5\right)$$

$$= \ln x + \frac{1}{3}\ln y^4 - \frac{1}{3}\ln z^5$$

$$= \ln x + \frac{4}{3}\ln y - \frac{5}{3}\ln z$$

(54) p = selling price (\$)
x = demand (# sold/day)

$p = p_0 e^{-ax}$
solve for x.

$e^{-ax} = \frac{p}{p_0}$ / ln

$\ln e^{-ax} = \ln \frac{p}{p_0}$

$-ax = \ln \frac{p}{p_0}$

$\boxed{x = \frac{-1}{a} \ln \frac{p}{p_0}} = \frac{1}{a} \ln \frac{p_0}{p}$

$\boxed{4.6}$

(32) $e^x + 4e^{-x} = 5$

$e^x + \frac{4}{e^x} = 5$ / $\cdot e^x \neq 0$

$(e^x)^2 + 4 = 5e^x$

let $e^x = t$ $(t > 0)$

then:

$t^2 - 5t + 4 = 0$

$(t-1)(t-4) = 0$

$t = 1$ OR $t = 4$

$e^x = 1$ $\quad e^x = 4$

$x = 0$ $\quad \ln 4 = x$

$\boxed{x \in \{0, \ln 4\}}$

(35) $y = \frac{10^x + 10^{-x}}{2}$
solve for x.

$2y = 10^x + 10^{-x}$

$2y = 10^x + \frac{1}{10^x}$ / $\cdot 10^x \neq 0$

$2y(10^x) = (10^x)^2 + 1$

let $10^x = t$ $(t > 0)$

$2yt = t^2 + 1$

$t^2 - 2yt + 1 = 0$

$t = \frac{2y \pm \sqrt{(2y)^2 - 4}}{2}$

$t = \frac{2y \pm \sqrt{4(y^2-1)}}{2}$

$t = y \pm \sqrt{y^2-1}$

note that $y^2 - 1 < y^2$

$\Rightarrow \sqrt{y^2-1} < \sqrt{y^2}$

$\Rightarrow \sqrt{y^2-1} < |y|$

therefore $y \pm \sqrt{y^2-1} > 0$

$t = y \pm \sqrt{y^2-1}$

$10^x = y \pm \sqrt{y^2-1}$

$\boxed{x = \log(y \pm \sqrt{y^2-1})}$

(50) Definition:

$pH = -\log[H^+]$

where $[H^+]$ = hydrogen ion conc. in moles/liter

Given $1 < pH < 14$

$1 < -\log[H^+] < 14 \quad / \cdot (-1)$

$-1 > \log[H^+] > -14$

$\left.\begin{array}{l} -14 < \log[H^+] < -1 \\ f(x) = 10^x \text{ increasing} \end{array}\right\} \rightarrow$

$10^{-14} < 10^{\log[H^+]} < 10^{-1}$

$$\boxed{10^{-14} < [H^+] < 10^{-1}}$$

(60)

$T(n)$ = time to assemble the n^{th} item

T_1 = time for the 1st item

$T(n) = T_1 n^{-k}$, $k > 0$

(a) $\left.\begin{array}{l} T(2) = (0.80)\, T_1 \\ \text{also, } T(2) = T_1\, 2^{-k} \end{array}\right\} \Rightarrow$

$(0.80)\,\cancel{T_1} = \cancel{T_1}\, 2^{-k}$

$2^{-k} = 0.80 \quad / \log_2$

$\log_2 2^{-k} = \log_2 0.8$

$-k = \log_2 0.8$

$$\boxed{k = -\log_2 0.8}$$

(b) $n = 4$

$T(4) = T_1 (4)^{-(-\log_2 0.8)}$

$T(4) = T_1 (4)^{\log_2 0.8} =$

$= T_1 (2^2)^{\log_2 0.8}$

$= T_1 (2)^{2\log_2 0.8}$

$= T_1 (2)^{\log_2 0.64}$

$= T_1 (0.64) \qquad (b^{\log_b x} = x)$

$$\boxed{T(4) = 0.64 T_1}$$

(c) $T(2n) = T_1 (2n)^{-k}$

$T(n) = T_1 n^{-k}$

$T(2n) = T_1\, 2^{-k} n^{-k}$

$= (T_1 n^{-k})\, 2^{-k}$

$T(2n) = T(n)\, 2^{-k}$

$= T(n)\, 2^{\log_2 0.8}$

$$\boxed{T(2n) = 0.8\, T(n)}$$