Math 160

HOMEWORK # 2

SOLUTIONS - SELECTED EXERCISES

SECTION 2.5

(28) $P(3,-1) \in$ graph of f

$y = 2f(x) + 4$

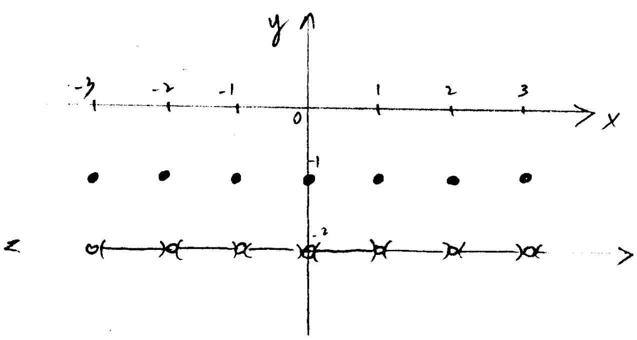
1st $\quad y = f(x) \qquad P(3,-1)$

2nd $\quad y = 2f(x)$ vertical stretch by a factor of 2 $\qquad (3,-2)$

3rd $\quad y = 2f(x) + 4$ shift up 4 $\qquad (3,2)$

So the corresponding point on the graph of $y = 2f(x)+4$ is $(3,2)$

(48) $f(x) = \begin{cases} -1 & \text{if } x = \text{integer} \\ -2 & \text{if } x \neq \text{integer} \end{cases}$



SECTION 2.6

(38) $D(h) = -0.078h^2 + 3.811h - 32.433$

quadratic equation in h

its graph is a parabola that opens downward, therefore the maximum occur at the vertex $V(h_v, D_v)$

$$h_v = \frac{-b}{2a} = \frac{-3.811}{2(-0.078)} \approx 24.43 \text{ km}$$

The altitude at which the density of ozone is greatest is 24.43 km.

(45) 1000 ft fence

a) Perimeter = 1000 $\Rightarrow$

$3x + 4y = 1000$

$4y = -3x + 1000$

$y = -\frac{3}{4}x + 250$

b) Area = A

$A = xy$

$A = x\left(-\frac{3}{4}x + 250\right)$

$A = -\frac{3}{4}x^2 + 250x$

c) $A = -\frac{3}{4}x^2 + 250x$ quadratic equation in x; its graph is a parabola that opens down, therefore the maximum occurs at the vertex $V(x_v, A_v)$

$$x_v = \frac{-b}{2a} = \frac{-250}{2(-\frac{3}{4})} = \frac{500}{3} = 166\tfrac{2}{3} \text{ ft}$$

Then $y = -\frac{3}{4}x + 250$

$y = -\frac{3}{4} \cdot \frac{500}{3} + 250 = 125$ ft

The dimensions that will max. the area are $166\frac{2}{3}$ ft by 125 ft.

(49) Vertex = (0,10)

a) $y = a(x-x_v)^2 + y_v$

$y = ax^2 + 10$

(200, 90) ∈ graph

x = 200, y = 90

$90 = a(200)^2 + 10 \Rightarrow a = \frac{1}{500}$

so $y = \frac{1}{500}x^2 + 10$

b) The cables are spaced 40 ft apart. (400 ÷ 10)

Letting x = 40, 80, 120, 160 gives us

Total length of

$10 + 2\left(\frac{66}{5} + \frac{114}{5} + \frac{194}{5} + \frac{306}{5}\right) =$

= 282 ft

(55) Let y = number of $5 decreases in the monthly charge

$R(y) = (\#\text{ of customers}) \cdot (\text{mo. charge per customer})$

$R(y) = (8000 + 1000y)(50 - 5y)$

Let x = monthly charge

$x = 50 - 5y$

$5y = 50 - x$

$y = \frac{50-x}{5}$

Substitute into R(y) =>

$R(x) = \left[8000 + 1000\left(\frac{50-x}{5}\right)\right]x$

$R(x) = \left(8000 + 200(50-x)\right)x$

$R(x) = 200x(90-x)$

b) $R(x) = -200x^2 + 18000x$

quadratic equation in x

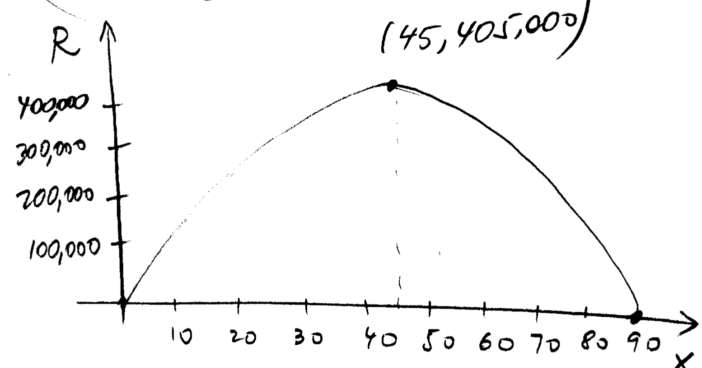
$V(x_v, R_v)$ $\quad x_v = \frac{-b}{2a} = \frac{-18000}{2(-200)}$

$x_v = 45$

then $R_v = 405,000$

V(45, 405,000)

(OR, to find the vertex, note that the x-int are at x = 0, x = 90 and x_v must be halfway between the x-int)



$x_v = 45$ $/mo, then $y = \frac{50-45}{5}$

y = 1 (one 5$ decrease)

Then there are 8000 + 1000 = = 9000 customers

Revenue = 9000($45) = 405,000

SECTION 2.7

(26) $f(x) = \sqrt{3-x}$

Domain: $3 - x \geq 0$
$3 \geq x$
$x \leq 3$

$D_f = (-\infty, 3]$

$g(x) = \sqrt{x+2}$

Domain: $x + 2 \geq 0$
$x \geq -2$

$D_g = [-2, \infty)$

a) $(f \circ g)(x) = f(g(x))$
$= f(\sqrt{x+2})$
$= \sqrt{3 - \sqrt{x+2}}$

$\begin{cases} x \in D_g \\ \text{and} \\ 3 - \sqrt{x+2} \geq 0 \end{cases} \iff \begin{cases} x \in [-2, \infty) \\ \text{and} \\ x \in (-\infty, 7] \end{cases}$

$3 - \sqrt{x+2} \geq 0$
$3 \geq \sqrt{x+2}$
$9 \geq x+2$
$7 \geq x$
$x \leq 7$

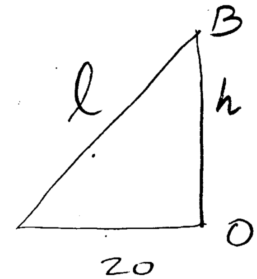
Therefore,
$x \in [-2, \infty) \cap (-\infty, 7]$
$x \in [-2, 7]$
Domain of $f \circ g$

(41) f = odd iff $f(-x) = -f(x)$
g = even iff $g(-x) = g(x)$

$(fg)(-x) = f(-x)g(-x)$
$= (-f(x))\, g(x)$
$= -(f(x)g(x))$
$= -(fg)(x)$

Therefore, fg is odd.

(49) $v = 5$ ft/sec
Let h = altitude
t = time



Let l = length of rope

when $t=0$, $l = 20$
At time t, $l = 20 + 5t$

ΔAOB: $20^2 + h^2 = l^2$
$400 + h^2 = (20+5t)^2$
$h^2 = (20+5t)^2 - 400$
$h^2 = 400 + 200t + 25t^2 - 400$
$h^2 = 25t^2 + 200t$
$h^2 = 25(t^2 + 8t)$
$h = \sqrt{25(t^2+8t)}$
$h = 5\sqrt{t^2 + 8t}$

SECTION 3.1

(22) $f(x) = \frac{-1}{8}(x+4)(x-2)(x-6)$

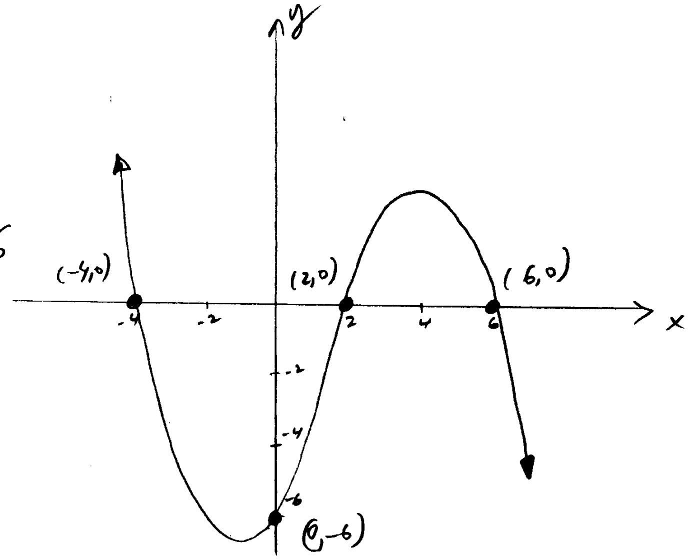
x	$-\infty$		-4		0		2		6		$-\infty$
$f(x)$	∞	$+$	0 $m=1$	$-$	-6	$-$	0 $m=1$	$+$	0 $m=1$	$-$	$-\infty$

Domain: $x \in \mathbb{R}$

x-n: $x=-4$, $x=2$, $x=6$
all of multiplicity 1

y-n: $x=0$, $y = \frac{-1}{8}(4)(-2)(-6) = -6$

$x \to \infty$, $y \to -\infty$
$x \to -\infty$, $y \to \infty$



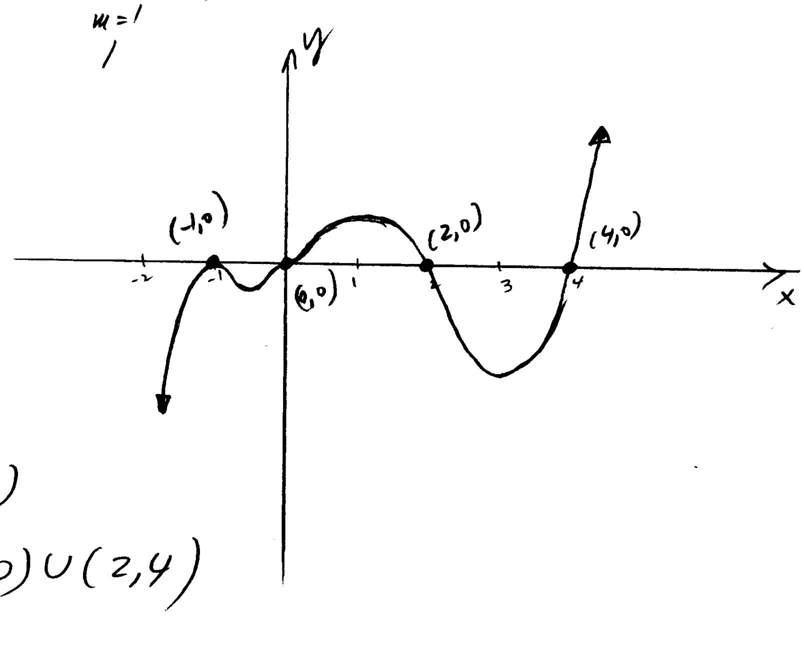
$f(x) > 0$ iff $x \in (-\infty, -4) \cup (2, 6)$
$f(x) < 0$ iff $x \in (-4, 2) \cup (6, \infty)$

(28) $f(x) = x^3(x+1)^2(x-2)(x-4)$

x	$-\infty$		-1		0		2		4		∞
$f(x)$	$-\infty$	$-$	0 $m=2$	$-$	0 $m=3$	$+$	0 $m=1$	$-$	0 $m=1$	$+$	∞

x-n: $x=0$ $m=3$
$x=-1$ $m=2$
$x=2$ $m=1$
$x=4$ $m=1$

$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to -\infty$



$f(x) > 0$ iff $x \in (0, 2) \cup (4, \infty)$
$f(x) < 0$ iff $x \in (-\infty, -1) \cup (-1, 0) \cup (2, 4)$