2.6 Mathematical Models: Building Functions

Exercise 1 (#2 page 109) Let $P=(x, y)$ be a point on the graph of $y=x^2-8$.

a) Express the distance d from P to the point $(0,-1)$ as a function of x.

b) What is d if $x=0$?

c) What is d if $x=-1$?

d) Use a graphing utility to graph $d=d(x)$.

e) For what values of x is d smallest?

Exercise 2 (#4 page 109) Let $P=(x, y)$ be a point on the graph of $y=\frac{1}{x}$.

a) Express the distance d from P to the origin as a function of x.

b) Use a graphing utility to graph $d=d(x)$.

c) For what values of x is d smallest?

Exercise 3 (#6 page 109) A right triangle has one vertex on the graph of $y=9-x^2$, $x>0$, at (x, y), another at the origin, and the third on the positive x-axis at $(x, 0)$. Express the area A of the triangle as a function of x.

Exercise 4 (#8 page 109) A rectangle is inscribed in a semicircle of radius 2. Let $P=(x, y)$ be a point in quadrant I that is a vertex of the rectangle and is on the circle.

a) Express the area A of the rectangle as a function of x.

b) Express the perimeter p of the rectangle as a function of x.

c) Graph $A=A(x)$. For what values of x is A largest?

d) Graph $p=p(x)$. For what values of x is p largest?

Exercise 5 (#12 page 110) A wire 10 meters long is to be cut into two pieces. One piece will be shaped as an equilateral triangle, and the other piece will be shaped as a circle.

a) Express the total area A enclose by the pieces of wire as a function of the length x of a side of the equilateral triangle.

b) What is the domain of A?

c) Graph $A=A(x)$. For what values of x is A smallest?

Exercise 6 (#18 page 110) Two cars leave an intersection at the same time. One is headed south at a constant speed of 30 miles per hour, and the other is headed west at a constant speed of 40 miles per hour. Build a model that expresses the distance d between the cars as a function of the time.

Exercise 7 (#20 page 110) Inscribe a right circular cylinder of height *h* and radius *r* in a sphere of fixed radius *R*. Express the volume *V* of the cylinder as a function of *h*.

Exercise 8 (#21 page 110) Inscribe a right circular cylinder of height *h* and radius *r* in a cone of fixed radius *R* and fixed height *H*. Express the volume *V* of the cylinder as a function of *r*.

Exercise 9 (#22 page 111) MetroMedia Cable is asked to provide service to a customer whose house is located 2 miles from the road along which the cable is buried. The connection box for the cable is located 5 miles down the road.

a) If the installation cost is $500 per mile along the road and $700 per mile off the road, build a model that expresses the total cost C of installation as a function of the distance x (in miles) from the connection box to the point where the cable installation turns off the road.
b) Compute the cost if $x = 1$ mile.
c) Compute the cost if $x = 3$ miles.
d) Graph the function $C = C(x)$.
e) What values of x result in the least cost?

Exercise 10 (#24 page 111) Water is poured into a container in the shape of a right circular cone with radius 4 feet and height 16 feet. Express the volume *V* of the water in the cone as a function of the height *h* of the water.

Answers:

1) b)7, c) $\sqrt{37}$, e) ± 2.55 ; **2)** c) ± 1 ; **3)** $A(x) = \frac{9}{2}x - \frac{1}{2}x^3$ **4)** a) $A(x) = 2x\sqrt{4-x^2}$, b) $P(x) = 4x + 2\sqrt{4-x^2}$,

c) 1.41, d) 1.79; **5)** b) 0<x<10/3, c) 2.08; **6)** $d(t) = 50t$; **6)** $A(x) = \frac{\sqrt{3}}{4}x^2 + \frac{(10-3x)^2}{4p}$

7) $V(h) = p\,h\left(R^2 - \frac{h^2}{4}\right)$; **8)** $V(t) = \frac{p\,H(R-r)r^2}{R}$; **9)** a) $C(x) = 500x + 700\sqrt{x^2 - 10x + 29}$, b) 3630.50,

c) 3479.90, e) 2.96 mi; **10)** $V(h) = \frac{p}{48}h^3$